P60

1 $= 2xe^{a^2+x^2}$

2 $= e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$

3 $= \dfrac{a}{ax+b}$

4 $= \dfrac{3x^2-2}{x^3-2x+6}$

5 $= \dfrac{3}{x}$

6 $= \dfrac{3\log^2 x}{x}$

7 $= \dfrac{1+\dfrac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}} = \dfrac{\sqrt{1+x^2}+x}{x+\sqrt{1+x^2}\left(\sqrt{1+x^2}\right)} = \dfrac{1}{\sqrt{1+x^2}}$

8 $= \dfrac{a-x}{a+x} * \dfrac{a-x-(a+x)(-1)}{(a-x)^2} = \dfrac{2a}{a^2-x^2}$

9 $= (\log e^x - \log(1+e^x))' = 1 - \dfrac{e^x}{1+e^x} = \dfrac{1}{1+e^x}$

10 $= \dfrac{a}{2} * \left(\dfrac{1}{a}e^{\frac{x}{a}} - \dfrac{-1}{a}e^{-\frac{x}{a}}\right) = \dfrac{1}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$

11 $y = x^{\frac{1}{\log x}}$ とおいて対数をとると $\log y = \dfrac{1}{\log x}\log x = 1$ $\dfrac{1}{y}\dfrac{dy}{dx} = 0$ $\dfrac{dy}{dx} = 0$

12 $y = x^{e^x}$ とおいて対数をとると $\log y = e^x \log x$

$\dfrac{1}{y}\dfrac{dy}{dx} = e^x \log x + \dfrac{e^x}{x}$ $\dfrac{dy}{dx} = x^{e^x}e^x\left(\log x + \dfrac{1}{x}\right)$

13 $= \dfrac{-1}{2\sqrt{1-x}}(x+x^2) + \sqrt{1-x} * (1+2x) = \dfrac{-x-x^2+2(1+x-2x^2)}{2\sqrt{1-x}} = \dfrac{2+x-5x^2}{2\sqrt{1-x}}$

14 $y = \dfrac{(x+1)^2}{(x+2)^3(x+3)^4}$ とおいて対数をとると $\log y = 2\log(x+1) - 3\log(x+2) - 4\log(x+3)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2}{x+1} - \dfrac{3}{x+2} - \dfrac{4}{x+3} = \dfrac{2(x+2)(x+3) - 3(x+1)(x+3) - 4(x+1)(x+2)}{(x+1)(x+2)(x+3)}$

$= \dfrac{2x^2+10x+12-3x^2-12x-9-4x^2-12x-8}{(x+1)(x+2)(x+3)} = \dfrac{-5x^2-14x-5}{(x+1)(x+2)(x+3)}$

$\dfrac{dy}{dx} = \dfrac{-(5x^2+14x+5)}{(x+1)(x+2)(x+3)} * \dfrac{(x+2)^2}{(x+2)^3(x+3)^4} = -\dfrac{(x+1)(5x^2+14x+5)}{(x+2)^4(x+3)^5}$

15 $= \dfrac{1}{2}\left(\log\dfrac{1+\sin x}{1-\sin x}\right)' = \dfrac{1}{2} * \dfrac{1-\sin x}{1+\sin x} * \dfrac{\cos x(1-\sin x) + (1+\sin x)\cos x}{(1-\sin x)^2}$

$= \dfrac{1}{2} * \dfrac{2\cos x}{(1+\sin x)(1-\sin x)} = \dfrac{\cos x}{1-\sin^2 x} = \dfrac{\cos x}{\cos^2 x} = \dfrac{1}{\cos x}$

16 $= -\sin x * e^{\cos x}\sin x + e^{\cos x} * \cos x = e^{\cos x}(\cos x - \sin^2 x)$

17

$$\frac{dx}{d\theta} = 3a\sin^2\theta\cos\theta \qquad \frac{dy}{d\theta} = 3a\cos^2\theta(-\sin\theta)$$

$$\frac{dy}{dx} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta} = \frac{-\cos\theta}{\sin\theta} = -\cot\theta$$

18

$$\frac{dx}{dt} = \frac{3a + 3at^3 - 9at^3}{(1+t^3)^2} = \frac{3a(1-2t^3)}{(1+t^3)^2} \qquad \frac{dy}{dt} = \frac{6at + 6at^4 - 9at^4}{(1+t^3)^2} = \frac{3a(2t-t^4)}{(1+t^3)^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3at(2-t^3)}{3a(1-2t^3)} = \frac{t(2-t^3)}{1-2t^3}$$

19

両辺を微分すると $e^y\frac{dy}{dx} = \sin(a+y) + x\cos(a+y)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{e^y - x\cos(a+y)} = \frac{\sin(a+y)}{e^y - \frac{e^y\cos(a+y)}{\sin(a+y)}} = \frac{\sin^2(a+y)}{e^y\{\sin(a+y) - \cos(a+y)\}}$$

20

両辺の対数をとると $\log x = \frac{x-y}{y}\log e = \frac{x-y}{y}$ $\qquad y\log x = x - y$

$y(\log x + 1) = x \qquad y = \frac{x}{\log x + 1}$

$$\frac{dy}{dx} = \frac{1 + \log x - \frac{x}{x}}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

21

両辺を微分すると $\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$