微分問題解答

P31

1	$=4ax^3+6x$	2	$=6ax-8b$
3	$=(a+b)x^{a+b-1}$	4	$=\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{3}x^{-\frac{2}{3}}-x^{-2}$
5	$=5(3x+4)+3(5x-6)=30x+2$		
6	$=12x^2(2x^2+1)+4x(4x^3+1)=40x^4+12x^2+4x$		
7	$=\frac{(b+2cx)x-(a+bx+cx^2)}{x^2}=\frac{bx+2cx^2-a-bx-cx^2}{x^2}=\frac{cx^2-a}{x^2}=c-\frac{a}{x^2}$		
8	$=\frac{x+1-x+1}{(x+1)^2}=\frac{2}{(x+1)^2}$		
9	$=\frac{-2x}{(x^2-1)^2}$	10	$=-(n-1)x^{-n}$
11	$y=x^{-\frac{1}{2}}+x^{\frac{1}{2}}$　となるから		$\frac{dy}{dx}=-\frac{1}{2}x^{-\frac{3}{2}}+\frac{1}{2}x^{-\frac{1}{2}}$
12	$y'=2x-2$　となるから（2，5）における 接線の方程式は　$y-5=(2\cdot 2-2)(X-2)=2X-4$　　$y-2X-1=0$ 法線の方程式は　$y-5=-\frac{1}{2\cdot 2-2}(X-2)=-\frac{1}{2}X+1$　　$y=-\frac{1}{2}X+6$		

P34

1	$=2(2x^3+1)6x^2=12x^2(2x^3+1)$		
2	$=n(a+bx^2)^{n-1}2bx=2nbx(a+bx)^{n-1}$		
3	$=\frac{5}{2}(1-x^2)^{\frac{3}{2}}(-2x)=-5x(1-x^2)^{\frac{3}{2}}$	4	$=\frac{a}{2\sqrt{ax+b}}$
5	$=m(a+x)^{m-1}(b+x)^n+n(a+x)^m(b+x)^{n-1}$ $=(a+x)^{m-1}(b+x)^{n-1}\{(m+n)x+na+mb\}$		
6	$=\sqrt{a-x}-\frac{a+x}{2\sqrt{a-x}}=\frac{a-3x}{2\sqrt{a-x}}$	7	$=\frac{b}{a}\frac{-2x}{2\sqrt{a^2-x^2}}=-\frac{bx}{a\sqrt{a^2-x^2}}$
8	$y=\sqrt{\frac{1+x^2}{1-x^2}}$　　とおくと　　$\log y=\frac{1}{2}\log\frac{1+x^2}{1-x^2}$ 微分すると　$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2}*\frac{1-x^2}{1+x^2}*\frac{2x(1-x^2)-(1+x^2)(-2x)}{(1-x^2)^2}=\frac{1}{2}*\frac{4x}{(1+x^2)(1-x^2)}$ よって　　$\frac{dy}{dx}=\frac{2x}{(1+x^2)^{\frac{1}{2}}(1-x^2)^{\frac{3}{2}}}$		

9	$= \dfrac{-2x * x^{\frac{1}{2}} - (x^2+1) * \frac{1}{2}x^{-\frac{1}{2}}}{(x^2+1)^2 x} = -\dfrac{5x^2+1}{2x^{\frac{3}{2}}(x^2+1)^2}$
10	$= \dfrac{1+\sqrt{1-x^2} - x * \frac{-x}{\sqrt{1-x^2}}}{\left(1+\sqrt{1-x^2}\right)^2} = \dfrac{\sqrt{1-x^2}+1-x^2+x^2}{\left(1+\sqrt{1-x^2}\right)^2\sqrt{1-x^2}} = \dfrac{1}{\left(1+\sqrt{1-x^2}\right)\sqrt{1-x^2}}$
11	$= \dfrac{1}{2}\left(1+\sqrt{1+x^2}\right)^{-\frac{1}{2}} * \dfrac{x}{\sqrt{1+x^2}} = \dfrac{x}{2\left(1+\sqrt{1+x^2}\right)^{\frac{1}{2}}\sqrt{1+x^2}}$
12	$= (a^2+3x^2)\sqrt{a^2-x^2} + (a^2x+x^3)\dfrac{-x}{\sqrt{a^2-x^2}} = \dfrac{a^4+2a^2x^2-3x^4-a^2x^2-x^4}{\sqrt{a^2-x^2}} = \dfrac{a^4+a^2x^2-4x^4}{\sqrt{a^2-x^2}}$

P43

1	$= \dfrac{a}{\cos^2(ax+b)} = a\sec^2(ax+b)$	2	$= -3\sin 3x$
3	$= -\dfrac{4x}{\sin^2(2x^2+3)} = -4x\csc^2(2x^2+3)$	4	$= 2\sec 2x\ \dfrac{\sin 2x}{\cos^2 2x}2 = 4\dfrac{\sin 2x}{\cos^3 2x}$
		5	$= 3\sin^2 x\ \cos^2 x - \sin^4 x$
6	$= \dfrac{1}{\cos^2\sqrt{1-x}}\dfrac{-1}{2\sqrt{1-x}} = -\dfrac{1}{2\sqrt{1-x}\ \cos^2\sqrt{1-x}}$	7	$= 2\sin 5x\cos 5x\ * 5 = 5\sin 10x$
8	$= \dfrac{-\sin x\ (1-\cos x) - (1+\cos x)\sin x}{(1-\cos x)^2} = \dfrac{-2\sin x}{(1-\cos x)^2}$		
9	$= \dfrac{-1}{\sin^2 x}\sin^{-1}x + \cot x\dfrac{1}{\sqrt{1-x^2}}$	10	$= \dfrac{2ax}{1+a^2x^2}$
11	$= x' = 1$		
12	$= \dfrac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}\ \dfrac{2-2x^2-2x(-2x)}{(1-x^2)^2} = \dfrac{2+2x}{1-2x^2+x^4+4x^2} = \dfrac{2(1+x^2)}{(1+x^2)^2} = \dfrac{2}{1+x^2}$		
13	$= \dfrac{1}{\sqrt{1-\frac{(x+1)^2}{2}}} * \dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2-(x+1)^2}} = \dfrac{1}{\sqrt{1-2x-x^2}}$		
14	$= \dfrac{1}{\frac{x^2+1}{x^2-1}\sqrt{\left(\frac{x^2+1}{x^2-1}\right)^2-1}}\ \dfrac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2} = \dfrac{-2}{x^2+1}$		
15	$= \dfrac{1}{2}\dfrac{-2x}{\sqrt{a^2-x^2}} + a\dfrac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\dfrac{1}{a} = \dfrac{-x}{\sqrt{a^2-x^2}} + \dfrac{a}{\sqrt{a^2-x^2}} = \dfrac{\sqrt{a-x} * \sqrt{a-x}}{\sqrt{(a-x)(a+x)}} = \sqrt{\dfrac{a-x}{a+x}}$		
16	$= \dfrac{1}{1+\left(\frac{4\sin x}{3+5\cos x}\right)^2} * \dfrac{12\cos x + 20\cos^2 x - (-20\sin^2 x)}{(3+5\cos x)^2} = \dfrac{12\cos x+20}{9+15\cos x+25\cos^2 x+16\sin^2 x}$ $= \dfrac{12\cos x+20}{9+15\cos x+25\cos^2 x+16-16\cos^2 x} = \dfrac{4(5+3\cos x)}{25+15\cos x+9\cos^2 x} = \dfrac{4(5+3\cos x)}{(5+3\cos x)^2} = \dfrac{4}{5+3\cos x}$		
17	$= \dfrac{-x}{\sqrt{1-x^2}}\sin^{-1}x + \dfrac{\sqrt{1-x^2}}{\sqrt{1-x^2}} - 1 = \dfrac{-x}{\sqrt{1-x^2}}\sin^{-1}x$		

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$$= \frac{\left(\sin^{-1}x + \frac{x}{\sqrt{1-x^2}}\right)\sqrt{1-x^2} - x\sin^{-1}x * \frac{-x}{\sqrt{1-x^2}}}{1-x^2} = \frac{(1-x^2)\sin^{-1}x + x\sqrt{1-x^2} + x^2\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}$$

$$= \frac{\sin^{-1}x + x\sqrt{1-x^2}}{(1-x^2)^{\frac{3}{2}}} = \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} + \frac{x}{1-x^2}$$

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$$= -\frac{1}{\sqrt{1-\left(\frac{1+2\cos x}{2+\cos x}\right)^2}} * \frac{-2\sin x(2+\cos x) + (1+2\cos x)\sin x}{(2+\cos x)^2}$$

$$= -\frac{-3\sin x}{\sqrt{4+4\cos x+\cos^2 x - 1 - 4\cos x - 4\cos^2 x} * (2+\cos x)} = \frac{3\sin x}{\sqrt{3-3\cos^2 x} * (2+\cos x)}$$

$$= \frac{3\sin x}{\sqrt{3}\sqrt{\sin^2 x} * (2+\cos x)} = \frac{\sqrt{3}\sin x}{(2+\cos x)|\sin x|}$$

$\sin x > 0$ とき $= \frac{\sqrt{3}}{2+\cos x}$　　$\sin x < 0$ とき $= \frac{-\sqrt{3}}{2+\cos x}$

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$y = \frac{(\sin nx)^m}{(\cos mx)^n}$　　とおいて両辺の対数をとると

$$\log y = m\log(\sin nx) - n\log(\cos mx)$$

微分すると　$\frac{1}{y}\frac{dy}{dx} = \frac{mn\cos nx}{\sin nx} - \frac{-mn\sin mx}{\cos mx}$

$$\frac{dy}{dx} = \frac{mn\cos nx * \cos mx + mn\sin nx * \sin mx}{\sin nx * \cos mx} * \frac{\sin nx^m}{\cos mx^n}$$

$$= mn(\cos nx * \cos mx + \sin nx * \sin mx)\frac{(\sin nx)^{m-1}}{(\cos mx)^{n+1}}$$

$$= mn\cos(nx - mx)\frac{(\sin nx)^{m-1}}{(\cos mx)^{n+1}}$$　　（三角法公式17によろ）