定積分解答

P175

1	$=\left[\frac{x^4}{4}\right]_0^1=\frac{1}{4}$	4	$=[\log\lvert x\rvert]_{-2}^{-1}=\log 1-\log 2=-\log 2$
2	$=\int_0^{\frac{\pi}{2}}\frac{1}{2}(\cos 2x+1)dx=\left[\frac{1}{2}\left(\frac{1}{2}\sin 2x+x\right)\right]_0^{\frac{\pi}{2}}=\frac{1}{4}(\sin\pi-\sin 0)+\frac{1}{2}\left(\frac{\pi}{2}-0\right)=\frac{\pi}{4}$		
3	$=\int_0^4\frac{1}{(x-2)^2+2^2}dx=\frac{1}{2}\left[\tan^{-1}\frac{x-2}{2}\right]_0^4=\frac{1}{2}\left\{\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right\}=\frac{\pi}{4}$		

P179

1	$=[\log\lvert x\rvert]_1^e=\log e-\log 1=1-0=1$
2	$=\frac{1}{2}\int_0^1(x^2-2x+2)(x^2-2x+2)'dx=\left[\frac{1}{2}\cdot\frac{1}{2}(x^2-2x+2)^2\right]_0^1=\frac{1}{4}(1-4)=-\frac{3}{4}$
3	$=\frac{1}{a}\left[\tan^{-1}\frac{x}{a}\right]_0^a=\frac{1}{a}(\tan^{-1}1-\tan^{-1}0)=\frac{1}{a}\left(\frac{\pi}{4}-0\right)=\frac{\pi}{4a}$
4	$=\int_0^2\left(x^2-x+1-\frac{1}{x+1}\right)dx=\left[\frac{x^3}{3}-\frac{x^2}{2}+x-\log\lvert x+1\rvert\right]_0^2=\frac{8}{3}-\frac{4}{2}+2-(\log 3-\log 1)=\frac{8}{3}-\log 3$
5	$=\frac{1}{2}\int_2^3\frac{(1+x^2)'}{1+x^2}dx=\frac{1}{2}[\log(1+x^2)]_2^3=\frac{1}{2}(\log 10-\log 5)=\frac{1}{2}\log 2$
6	$\sin x=t$　とおくと　$dx=\frac{dx}{\cos x}$　　$=\int_0^1\frac{\cos x}{1+t^2}\frac{dt}{\cos x}=[\tan^{-1}t]_0^1=\frac{\pi}{4}-0=\frac{\pi}{4}$
7	
8	$\tan\frac{x}{2}=t$　とおくと　$x=2\tan^{-1}t$　$dx=\frac{2}{1+t^2}$
	$=\int_0^1\frac{1}{\frac{2t}{1+t^2}}\cdot\frac{2dt}{1+t^2}=\int_0^1\frac{1}{t}dt=[\log\lvert t\rvert]_0^1=0$
9	$=\int_0^1\frac{1}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx=\frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{2x-1}{\sqrt{3}}\right]_0^1=\frac{2}{\sqrt{3}}\left(\tan^{-1}\frac{1}{\sqrt{3}}-\tan^{-1}\frac{-1}{\sqrt{3}}\right)=\frac{2}{\sqrt{3}}\left(\frac{\pi}{6}-\left(-\frac{\pi}{6}\right)\right)=\frac{2\pi}{3\sqrt{3}}$
10	$=\frac{1}{2}\int_0^1\frac{2x+1+3}{x^2+x+1}dx=\frac{1}{2}[\log\lvert x^2+x+1\rvert]_0^1+\frac{3}{2}\int_0^1\frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$ $=\frac{1}{2}(\log 3-\log 1)+\frac{3}{2}\frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{2x+1}{\sqrt{3}}\right]_0^1=\frac{1}{2}\log 3+\sqrt{3}\left(\tan^{-1}\sqrt{3}-\tan^{-1}\frac{1}{\sqrt{3}}\right)$ $=\frac{1}{2}\log 3+\sqrt{3}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)=\frac{1}{2}\log 3+\frac{\sqrt{3}}{6}\pi$

11	$= \frac{1}{2}\int_0^1 \frac{2x}{\sqrt{1+x^2}}dx = \frac{1}{2}\left[2\sqrt{1+x^2}\right]_0^1 = \sqrt{2}-1$
12	$= -\frac{1}{2}\int_0^1 \frac{-2}{\sqrt{3-2x}}dx = -\frac{1}{2}\left[2\sqrt{3-2x}\right]_0^1 = -1+\sqrt{3} = \sqrt{3}-1$

P181

5	$= \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a = \frac{a}{2}\left(\frac{\pi}{2}-0\right) = \frac{\pi a^2}{4}$ （P138-11参照）
6	$\sqrt{1-x}=t$ とおくと $x=1-t^2$ $dx=-2tdt$ $=\int_1^0 (1-2t^2+t^4)t\cdot(-2t)dt = -2\int_1^0 (t^2-2t^4+t^6)dt$ $=-2\left[\frac{t^3}{3}-\frac{2}{5}t^5+\frac{1}{7}t^7\right]_1^0 = -2(-1)\left(\frac{35-42+15}{105}\right)\frac{16}{105}$
9	$= -\int_0^{\frac{\pi}{2}}(1-\cos^2 x)^2\cos^4 x(\cos x)'dx = -\int_0^{\frac{\pi}{2}}(\cos^4 x - 2\cos^6 x + \cos^8 x)(\cos x)'dx$ $=\left[\frac{\cos^5 x}{5}-\frac{2\cos^7 x}{7}+\frac{\cos^9 x}{9}\right]_{\frac{\pi}{2}}^0 = \frac{1}{5}-\frac{2}{7}+\frac{1}{9}=\frac{8}{315}$
10	$=\int_0^{\frac{\pi}{2}}\sin^4 x(1-\sin^2 x)\cos x dx = \int_0^{\frac{\pi}{2}}(\sin^4 x - \sin^6 x)(\sin x)'dx = \left[\frac{\sin^5 x}{5}-\frac{\sin^7 x}{7}\right]_0^{\frac{\pi}{2}} = \frac{1}{5}-\frac{1}{7}=\frac{2}{35}$

P186

1	$\sqrt{x}=t$ とおくと $x=t^2$ $dx=2tdt$ $=\int_0^2\frac{2t}{1+t}dt = 2\int_0^2\frac{1+t-1}{1+t}dt = 2\int_0^2\left(1-\frac{1}{1+t}\right)dt = 2[t-\log(1+t)]_0^2 = 4-2\log 3$
2	$\sqrt{2+4x}=t$ とおくと $x=\frac{1}{4}(t^2-2)$ $dx=\frac{1}{2}tdt$ $=\frac{1}{4}\int_{\sqrt2}^{3\sqrt2}\frac{t^2-2}{t}\cdot\frac{1}{2}tdt = \frac{1}{8}\int_{\sqrt2}^{3\sqrt2}(t^2-2)dt = \frac{1}{8}\left[\frac{t^3}{3}-2t\right]_{\sqrt2}^{3\sqrt2} = \frac{1}{8}\left(18\sqrt2-\frac{2\sqrt2}{3}-6\sqrt2+2\sqrt2\right)=\frac{5}{3}\sqrt2$
3	$1+2x=t$ とおくと $x=\frac{t}{2}$ $dx=\frac{dt}{2}$ $=\int_1^3\frac{t-1}{2t^3}\cdot\frac{1}{2}dt = \frac{1}{4}\int_1^3\frac{t-1}{t^3}dt = \frac{1}{4}\int_1^3\left(\frac{1}{t^2}-\frac{1}{t^3}\right)dt = \frac{1}{4}\left[-\frac{1}{t}+\frac{1}{2t^2}\right]_1^3 = \frac{1}{8}\left[\frac{1-2t}{t^2}\right]_1^3=\frac{1}{18}$
4	$x^{\frac{1}{4}}=t$ とおくと $x=t^4$ $dx=4t^3dt$ $=\int_0^2\frac{t}{1+t^2}\cdot 4t^3dt = 4\int_0^2\left(t^2-1+\frac{1}{1+t^2}\right)dt = 4\left[\frac{t^3}{3}-t+\tan^{-1}t\right]_0^2=\frac{8}{3}+4\tan^{-1}2$
5	

6	$\sqrt{x+3}=t$ とおくと $x=t^2-3$ $dx=2tdt$ $=\int_2^3 \frac{t^2-3}{t}\cdot 2tdt=2\int_2^3 (t^2-3)dt=\left[\frac{2}{3}t^3-6t\right]_2^3=18-18-\frac{16}{3}+12=\frac{30}{3}$
7	$\tan\frac{x}{2}=t$ とおくと $x=2\tan^{-1}t$ $dx=\frac{2}{1+t^2}$ $=\int_0^1 \frac{1}{1+\frac{2t}{1+t^2}}\cdot\frac{1}{1+t^2}dt=\int_0^1\frac{2}{(t+1)^2}dt=\left[-\frac{2}{t+1}\right]_0^1=\left[\frac{2}{t+1}\right]_1^0=2-1=1$
8	$\tan\frac{x}{2}=t$ とおくと $x=2\tan^{-1}t$ $dx=\frac{2}{1+t^2}$ $=\int_1^\infty \frac{1}{1-\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}dt=\int_1^\infty\frac{1}{t^2}dt=\left[-\frac{1}{t}\right]_1^\infty=-0+1=1$
9	$f(x)=\frac{x^2}{2}$ とおくと $f(x)'=x$ $g(x)=\log x$ とおくと $g(x)'=\frac{1}{x}$ $=\left[\frac{1}{2}x^2\log x\right]_0^1-\frac{1}{2}\int_0^1 xdx=0-\frac{1}{2}\left[\frac{1}{2}x^2\right]_0^1=0-\frac{1}{4}=-\frac{1}{4}$
10	$f(x)=\tan^{-1}x$ とおくと $f(x)'=\frac{1}{1+x^2}$ $g(x)=\frac{x^3}{3}$ とおくと $g(x)'=x^2$ $=\left[\frac{x^3}{3}\tan^{-1}x\right]_0^1-\frac{1}{3}\int_0^1\frac{x^3}{1+x^2}dx=\frac{1}{3}\cdot\frac{\pi}{4}-\frac{1}{3}\int_0^1\left(x-\frac{1}{2}\cdot\frac{2x}{1+x^2}\right)dx$ $=\frac{\pi}{12}-\frac{1}{3}\left[\frac{x^2}{2}-\frac{1}{2}\log\lvert 1+x^2\rvert\right]_0^1=\frac{\pi}{12}-\frac{1}{6}+\frac{1}{6}\log 2$