P157

1 $\sqrt{1-x}=t$　とおくと　$dx=2tdt$

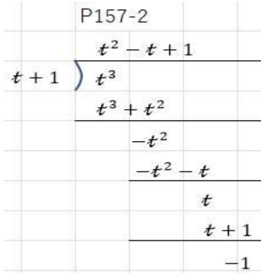
$$=\sqrt{\frac{2t}{t^2+1+t}}dt=\int\frac{2t+1-1}{t^2+t+1}dt=\log|t^2+t+1|-\int\frac{1}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dt$$

$$=\log|x-1+\sqrt{x-1}+1|-\frac{1}{\frac{\sqrt{2}}{2}}\tan^{-1}\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\log|x+\sqrt{x-1}|-\frac{2}{\sqrt{3}}\tan^{-1}\frac{2\sqrt{x-1}+1}{\sqrt{3}}$$

2 $\sqrt[6]{x}=t$　とおくと　$dx=6t^5dt$

$$=6\int\frac{t^5dt}{t^3+t^2}=6\int\frac{t^3}{t+1}dt=6\int\left(t^2-t+1-\frac{1}{t+1}\right)dt$$

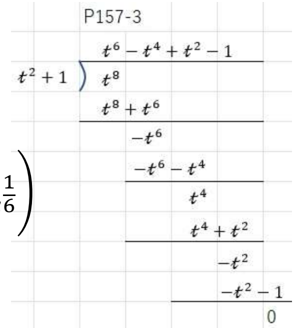
$$=6\left\{\frac{t^3}{3}-\frac{t^2}{2}+t-\log|t+1|\right\}=2x^{\frac{1}{2}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left|y^{\frac{1}{6}}+1\right|$$



3 $\sqrt[6]{x}=t$　とおくと　$dx=6t^5dt$

$$=6\int\frac{t^3}{1+t^2}6t^5\,dt=6\int\left(t^6-t^4+t^2-1+\frac{1}{t^2+1}\right)dt$$

$$=6\left(\frac{t^7}{7}-\frac{t^5}{5}+\frac{t^3}{3}-t+\tan^{-1}t\right)=6\left(\frac{x^{\frac{7}{6}}}{7}-\frac{x^{\frac{5}{6}}}{5}+\frac{x^{\frac{1}{2}}}{3}-x^{\frac{1}{6}}+\tan^{-1}x^{\frac{1}{6}}\right)$$



4

5 $\sqrt[6]{1+x}=t$　とおくと　$dx=6t^5dt$

$$=\int\frac{t^6-1+t^3}{t^2}6t^5dt=6\int(t^9+t^6-t^3)dt=6\left(\frac{t^{10}}{10}+\frac{t^7}{7}-\frac{t^4}{4}\right)$$

$$=\frac{3}{5}(1+x)^{\frac{5}{3}}+\frac{6}{7}(1+x)^{\frac{7}{6}}-\frac{3}{2}(1+x)^{\frac{2}{3}}$$

6 $\sqrt[n]{1+x}=t$　とおくと　$nx^{n-1}dx=2tdt$

$$=\int\frac{1}{x\cdot t}\cdot\frac{2tdt}{nx^{n-1}}=\frac{2}{n}\int\frac{dt}{x^n}=\frac{2}{n}\int\frac{dt}{t^2-1}=\frac{2}{n}\cdot\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|=\frac{1}{n}\log\left|\frac{\sqrt{1+x^n}-1}{\sqrt{1+x^n}+1}\right|$$

7 $\sqrt{1-x^2}=t$　とおくと　$-2xdx=2tdt$　$dx=-\frac{t}{x}dt$

$$=\int\frac{x^3}{t}\cdot-\frac{t}{x}dt=-\int x^2dt=\int(t^2-1)dt=\frac{t^3}{3}-t=-\frac{\sqrt{1-x^2}}{3}(x^2+2)$$

8 $(x^3+1)^{\frac{1}{3}}=t$　とおくと　$3x^2dx=3t^2dt$　$dx=\frac{t^2}{x^2}dt$

$$=\int x^2t^5\frac{t^2}{x^2}dt=\int t^7dt=\frac{1}{8}t^8=\frac{1}{8}(x^3+1)^{\frac{8}{3}}$$

P159

14

$e^{x}=t$　とおくと　　$e^{x}dx=dt$　　$dx=\dfrac{dt}{t}$

$$=\int\frac{1}{at+b}\cdot\frac{dt}{t}=\frac{1}{b}\int\left(\frac{1}{t}-\frac{a}{at+b}\right)dt=\frac{1}{b}(\log t-\log|at+b|)$$

$$=\frac{1}{b}(\log e^{x}-\log|ae^{x}+b|)=\frac{1}{b}(x-\log|ae^{x}+b|)$$

15

$\tan\dfrac{x}{2}=t$　とおくと　$\dfrac{x}{2}=\tan^{-1}t$　　$dx=\dfrac{2}{1+t^{2}}dt$

$$\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}=\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^{2}\frac{x}{2}+\sin^{2}\frac{x}{2}}=\frac{2\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\dfrac{\sin^{2}\frac{x}{2}}{\cos^{2}\frac{x}{2}}}=\frac{2\tan\frac{x}{2}}{1+\tan^{2}\frac{x}{2}}=\frac{2t}{1+t^{2}}$$

$$\cos x=\cos^{2}\frac{x}{2}-\sin^{2}\frac{x}{2}=\frac{\cos^{2}\frac{x}{2}-\sin^{2}\frac{x}{2}}{\cos^{2}\frac{x}{2}+\sin^{2}\frac{x}{2}}=\frac{1-\dfrac{\sin^{2}\frac{x}{2}}{\cos^{2}\frac{x}{2}}}{1+\dfrac{\sin^{2}\frac{x}{2}}{\cos^{2}\frac{x}{2}}}=\frac{1-\tan^{2}\frac{x}{2}}{1+\tan^{2}\frac{x}{2}}=\frac{1-t^{2}}{1+t^{2}}$$

$$\int\frac{dx}{\sin x+\cos x}=\int\frac{1}{\dfrac{2t}{1+t^{2}}+\dfrac{1-t^{2}}{1+t^{2}}}\cdot\frac{2}{1+t^{2}}dt=2\int\frac{dt}{1+2t-t^{2}}=2\int\frac{dt}{2-(1-2t+t^{2})}$$

$$=2\int\frac{dt}{\sqrt{2}^{2}-(1-t)^{2}}=\frac{2}{2\sqrt{2}}\int\left(\frac{1}{\sqrt{2}-(1-t)}+\frac{1}{\sqrt{2}+(1-t)}\right)dt$$

$$=\frac{1}{\sqrt{2}}\{(\log\sqrt{2}-1+t)+\log(\sqrt{2}+1-t)\cdot(-1)\}=\frac{1}{\sqrt{2}}\log\frac{\sqrt{2}-1+t}{\sqrt{2}+1-t}=\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}-1+\tan^{-1}\frac{x}{2}}{\sqrt{2}+1-\tan^{-1}\frac{x}{2}}\right|$$

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$\tan\dfrac{x}{2}=t$　とおくと　$\dfrac{x}{2}=\tan^{-1}t$　　$dx=\dfrac{2}{1+t^{2}}dt$

$$=\int\frac{\dfrac{2t}{1+t^{2}}}{1+\dfrac{2t}{1+t^{2}}}\cdot\frac{2}{1+t^{2}}dt=4\int\frac{t}{(1+t^{2}+2t)(1+t^{2})}dt=\frac{4}{2}\int\left(\frac{1}{1+t^{2}}-\frac{1}{1+2t+t^{2}}\right)dt$$

$$=2\left(\tan^{-1}t+\frac{1}{t+1}\right)=2\tan^{-1}\tan\frac{x}{2}+\frac{2}{\tan^{-1}\frac{x}{2}+1}=2\cdot\frac{x}{2}+\frac{2}{1+\tan\frac{x}{2}}=x+\frac{2}{1+\tan\frac{x}{2}}$$

17

$$=\int\sin^{4}x\cdot\sin x\,dx=\int(1-\cos^{2}x)^{2}\sin x\,dx$$

$-\cos x=t$　とおくと　　$\sin x\,dx=dt$　　$dx=\dfrac{dt}{\sin x}$

$$=\int(1-t^{2})^{2}\sin x\cdot\frac{dt}{\sin x}=\int(1-2t^{2}+t^{4})dt=t-\frac{2}{3}t^{3}+\frac{1}{5}t^{5}=-\cos x+\frac{2}{3}\cos^{3}x-\frac{1}{5}\cos^{5}x$$

1	$e^{ax}=t$ とおくと $ae^{ax}dx=dt$ $dx=\dfrac{dt}{at}$ $=\int\left(t+\frac{1}{t}\right)^2\frac{dt}{at}=\frac{1}{a}\int\left(t+\frac{2}{t}+\frac{1}{t^3}\right)dt=\frac{1}{a}\left(\frac{t^2}{2}+2\log t-\frac{1}{2}t^{-2}\right)$ $=\frac{1}{a}\left(\frac{1}{2}e^{2ax}+2\log e^{ax}-\frac{1}{2}e^{-2ax}\right)=\frac{1}{2a}(e^{2ax}-e^{-2ax})+\frac{2}{a}\log e^{ax}=\frac{1}{2a}(e^{2ax}-e^{-2ax})+2x$
2	$e^x=t$ とおくと $e^x dx=dt$ $dx=\dfrac{dt}{t}$ $=\int\frac{1}{t+\frac{1}{t}}\cdot\frac{dt}{t}=\int\frac{dt}{t^2+1}=\tan^{-1}t=\tan^{-1}e^x$
3	$=\int\frac{2t}{t+1}\cdot\frac{dt}{t}=\int\frac{2}{t+1}dt=2\log(e^x+1)$
4	$=\frac{-1}{b}\int\frac{(a+b\cos x)'}{a+b\cos x}dx=-\frac{1}{b}\log\lvert a+b\cos x\rvert$
5	$f(x)=\log\left(x+\sqrt{1+x^2}\right)$ とおくと $f(x)'=\dfrac{1}{\sqrt{1+x^2}}$ $g(x)=\dfrac{x^2}{2}$ とおくと $g(x)'=x$ $=\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\int\frac{x^2+1-1}{\sqrt{1+x^2}}dx=\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\int\sqrt{1+x^2}dx+\frac{1}{2}\int\frac{1}{\sqrt{1+x^2}}dx$ 公式（22）により $\int\sqrt{1+x^2}dx=\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)$ であるから $=\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{4}\log\left(x+\sqrt{1+x^2}\right)+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)$ $=\frac{1}{4}(2x^2+1)\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{4}x\sqrt{1+x^2}$
6	$f(x)=\sin^{-1}x$ とおくと $f(x)'=\dfrac{1}{\sqrt{1-x^2}}$ $g(x)=\dfrac{x^3}{3}$ とおくと $g(x)'=x^2$ $=\frac{x^3}{3}\sin^{-1}x-\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}dx$ ここで $\sqrt{1-x^2}=t$ とおくと $1-t^2=x^2$ $dx=\dfrac{-t}{x}dt$ $=\frac{x^3}{3}\sin^{-1}x-\frac{1}{3}\int\frac{x^3}{t}\cdot\frac{-t}{x}dt=\frac{x^3}{3}\sin^{-1}x+\frac{1}{3}\int(1-t^2)dt$ $=\frac{x^3}{3}\sin^{-1}x+\frac{1}{3}\sqrt{1-x^2}-\frac{1}{9}\left(\sqrt{1-x^2}\right)^3$

7	$\tan\frac{x}{2}=t$ とおくと $\frac{x}{2}=\tan^{-1}t \quad dx=\frac{2}{1+t^2}dt$ $=\int\frac{1}{1+\frac{1-\tan^2 x}{1+\tan^2 x}}dx=\int\frac{1}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2}=\int\frac{2dt}{1+t^2+1-t^2}=t=\tan\frac{x}{2}$
	(別解) $=\int\frac{1-\cos x}{(1+\cos x)(1-\cos x)}dx=\int\frac{1-\cos x}{1-\cos^2 x}dx=\int\left(\frac{1}{\sin^2 x}-\frac{\cos x}{\sin^2 x}\right)dx$ $=-\cos x+\csc x=\frac{1-\cos x}{\sin x}=\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\tan\frac{x}{2}$
8	$=\int\left(\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}\right)dx=\tan x-\cot x$