

$$\begin{aligned}
 1 \quad &= \int \left(x + 3 + \frac{2x^2 + 4}{x^3 - x^2 - 4x + 4} \right) dx \\
 &= \frac{x^2}{2} + 3x + \int \frac{2x^2 + 4}{x^2(x-1) - 4(x-1)} dx \\
 &= \frac{x^2}{2} + 3x + \int \left(\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+2} \right) dx
 \end{aligned}$$

		P142例1		
		$x+3$		
$x^3 - x^2 - 4x + 4$)	$x^4 + 2x^3 - 5x^2 - 8x + 6$		
		$x^4 - x^3 - 4x^2 + 4x$		
		$3x^3 - x^2 - 12x + 16$		
		$3x^3 - 3x^2 - 12x + 12$		
		$2x^2 +$		4

$A(x-2)(x+2) + B(x-1)(x+2) + C(x-1)(x-2) = 2x^2 + 4$ となるから

$$\begin{aligned}
 x = 1 \quad &\text{とおくと} & -3A = 6 & & A = -2 \\
 x = 2 \quad &\text{とおくと} & 4B = 12 & & B = 3 \\
 x = -2 \quad &\text{とおくと} & 12C = 12 & & C = 1 \quad \text{したがって}
 \end{aligned}$$

$$= \frac{x^2}{2} + 3x + \log \left| \frac{(x-2)^3(x+2)}{(x-1)^2} \right|$$

$$2 \quad = \int \left\{ \frac{A}{x-1} + \frac{B}{x-3} + \frac{C}{(x-3)^2} \right\} dx$$

$$A(x-3)^2 + B(x-1)(x-3) + C(x-1) = x^2 + 6x + 1$$

$$\begin{aligned}
 x = 3 \quad &\text{とおくと} & 2C = 28 & & C = 14 \\
 x = 1 \quad &\text{とおくと} & 4A = 8 & & A = 2 \\
 18 + 3B - 14 = 1 & & 3B = -3 & & B = -1 \quad \text{したがって}
 \end{aligned}$$

$$= \int \left\{ \frac{2}{x-1} - \frac{1}{x-3} + \frac{14}{(x-3)^2} \right\} dx = \log \left| \frac{(x-1)^2}{x-3} \right| - \frac{14}{x-3}$$

$$3 \quad = \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \right) dx \quad \begin{aligned} A(x^2+x+1) + (x-1)(Bx+C) &= 1 \\ x = 1 \quad \text{とおくと} & 3A = 1 & A = \frac{1}{3} \end{aligned}$$

$$\frac{1}{3}x^2 + \frac{1}{3}x + \frac{1}{3} + Bx^2 + (C-B)x - C = 1 \quad \begin{aligned} \frac{1}{3} - C = 1 & & C = -\frac{2}{3} & & B = -\frac{1}{3} \end{aligned}$$

$$= \frac{1}{3} \log|x-1| - \frac{1}{3} \int \frac{x+2}{x^2+x+1} dx = \frac{1}{3} \log|x-1| - \frac{1}{3} \int \frac{12x+1+3}{2x^2+x+1} dx$$

$$= \frac{1}{3} \log|x-1| - \frac{1}{6} \int \frac{2x+1}{x^2+x+1} dx - \frac{1}{6} \int \frac{3}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$= \frac{1}{3} \log|x-1| - \frac{1}{6} \log|x^2+x+1| - \frac{3}{6} \frac{1}{\sqrt{3}} \tan^{-1} \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{6} \log \left| \frac{(x-1)^2}{x^2+x+1} \right| - \frac{1}{\sqrt{3}} \tan^{-1} \frac{2x+1}{\sqrt{3}}$$

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$$1 + x^2 = t \quad \text{とおくと} \quad dx = \frac{dt}{2x}$$

$$= \int \frac{1}{xt^2} \cdot \frac{dt}{2x} = \frac{1}{2} \int \frac{dt}{(t-1)t^2} = \frac{1}{2} \int \left(\frac{A}{t-1} + \frac{B}{t} + \frac{C}{t^2} \right) dt$$

$$At^2 + Bt(t-1) + C(t-1) = 1$$

$t = 1$ とすると $A = 1$
 $t = 0$ とすると $C = -1$
 $A + B = 0$ $B = -1$

したがって

$$= \frac{1}{2} \left(\log|t-1| - \log t + \frac{1}{t} \right) = \frac{1}{2} \left(\log \left| \frac{1+x^2-1}{1+x^2} \right| + \frac{1}{1+x^2} \right) = \frac{1}{2} \left(\log \frac{x^2}{1+x^2} + \frac{1}{1+x^2} \right)$$

P147

1	$= \int \left(1 + \frac{3}{x^2-4} \right) dx = x + \int \frac{3}{4} \left(\frac{1}{x-2} - \frac{1}{x+2} \right) dx$ $= x + \frac{3}{4} \log \left \frac{x-2}{x+2} \right $	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td colspan="2" style="text-align: right;">P147-1</td></tr> <tr><td style="width: 50%;"></td><td style="text-align: center;">1</td></tr> <tr><td style="border-right: 1px solid black; border-bottom: 1px solid black;">x^2-4</td><td style="border-bottom: 1px solid black;">x^2-1</td></tr> <tr><td style="border-right: 1px solid black;"></td><td>x^2-4</td></tr> <tr><td style="border-right: 1px solid black;"></td><td style="text-align: center;">3</td></tr> </table>	P147-1			1	x^2-4	x^2-1		x^2-4		3				
P147-1																
	1															
x^2-4	x^2-1															
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2	$= \int \left(\frac{A}{x-1} + \frac{B}{x-2} \right)$ $A(x-2) + B(x-1) = 2x-1$ <p style="text-align: right;"> $x = 1$ とすると $-A = 1$ $A = -1$ $x = 2$ とすると $B = 3$ </p> <p>したがって</p> $= \log \left \frac{(x-2)^3}{x-1} \right $															
3	$= \int \left(\frac{A}{x+4} + \frac{B}{x-2} \right) dx$ $A(x-2) + B(x+4) = x+7$ <p style="text-align: right;"> $x = -4$ とすると $-6A = 3$ $A = -\frac{1}{2}$ $x = 2$ とすると $6B = 9$ $B = \frac{3}{2}$ </p> <p>したがって</p> $= \frac{3}{2} \log x-2 - \frac{1}{2} \log x+4 = \log \left \frac{(x-2)^{\frac{3}{2}}}{(x+4)^{\frac{1}{2}}} \right $															
4	$= -\frac{1}{4} \int \left(\frac{1}{x^2} - \frac{1}{x^2-4} \right) dx = -\frac{1}{4} \int \frac{1}{x^2} - \frac{1}{4} \left(\frac{1}{x-2} - \frac{1}{x+2} \right) dx = \frac{1}{4x} + \frac{1}{16} \log \left \frac{x-2}{x+2} \right $															
5	$= \int \left(2x + 6 + \frac{14x+13}{(x+1)(x+2)} \right)$ $= x^2 - 6x + \int \left(\frac{A}{x+1} + \frac{B}{x+2} \right) dx$ <p style="text-align: right;"> $A + B = 14$ $2A + B = 13$ $A = -1$ $B = 15$ </p> $= x^2 - 6x + \log \left \frac{(x+2)^{15}}{x+1} \right $	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td colspan="2" style="text-align: right;">P147-5</td></tr> <tr><td style="width: 50%;"></td><td style="text-align: center;">2x+6</td></tr> <tr><td style="border-right: 1px solid black; border-bottom: 1px solid black;">x^2+3x+2</td><td style="border-bottom: 1px solid black;">$2x^3+1$</td></tr> <tr><td style="border-right: 1px solid black;"></td><td>$2x^3+6x^2+4x$</td></tr> <tr><td style="border-right: 1px solid black;"></td><td>$-6x^2-4x+1$</td></tr> <tr><td style="border-right: 1px solid black;"></td><td>$-6x^2-18x-12$</td></tr> <tr><td style="border-right: 1px solid black;"></td><td style="text-align: center;">14x+13</td></tr> </table>	P147-5			2x+6	x^2+3x+2	$2x^3+1$		$2x^3+6x^2+4x$		$-6x^2-4x+1$		$-6x^2-18x-12$		14x+13
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6	$= \frac{1}{a^2-b^2} \int \left(\frac{1}{x^2-b^2} - \frac{1}{x^2+a^2} \right) dx = \frac{1}{a^2-b^2} \left(\frac{1}{b} \tan^{-1} \frac{x}{b} - \frac{1}{a} \tan^{-1} \frac{x}{a} \right)$															

8	$x^2 + 1 = t$ とおくと $2x dx = dt$ $dx = \frac{dt}{2x}$ $= \int \frac{1}{x \cdot t} \frac{dt}{2x} = \frac{1}{2} \int \frac{dt}{t(t-1)} = \frac{1}{2} \int \left(\frac{1}{t-1} - \frac{1}{t} \right) dt = \frac{1}{2} \log \left \frac{t-1}{t} \right = \log \left(\frac{x^2}{x^2+1} \right)^{\frac{1}{2}} = \log \left \frac{x}{\sqrt{x^2+1}} \right $
9	$= \int \left(\frac{A}{x+1} + \frac{Bx+C}{x^2+4} \right) dx$ $A(x^2+4)(x+1)(Bx+C) = x$ $Ax^2 + 4A + Bx^2 + (B+C)x + C = x$ $A+B=0$ $A = -\frac{1}{5}$ $B = \frac{1}{5}$ $C = \frac{4}{5}$ $B+C=1$ $4A+C=0$ $= -\frac{1}{5} \int \frac{dx}{x+1} + \frac{1}{5} \int \frac{x+4}{x^2+4} dx = -\frac{1}{5} \log x+1 + \frac{1}{10} \int \frac{2x+8}{x^2+4} dx$ $= -\frac{1}{5} \log x+1 + \frac{1}{10} \log x^2+4 + \frac{8}{10} \int \frac{1}{x^2+2^2} dx = \frac{1}{10} \log \left \frac{x^2+4}{(x+1)^2} \right + \frac{2}{5} \tan^{-1} \frac{x}{2}$
10	$= \frac{1}{2} \int \frac{2x}{(x^2+4)^5} dx = \frac{1}{2} \cdot \frac{1}{-4} \cdot \frac{1}{(x^2+4)^4} = -\frac{1}{8(x^2+4)^4}$

P149

6	$\sqrt{1-x} = t$ とおくと $dx = -2t dt$ $= \int \frac{-2t dt}{(1-t^2)t} = 2 \int \frac{1}{t^2-1} dt = \int \left(\frac{1}{t-1} - \frac{1}{t+1} \right) dt = \log \left \frac{t-1}{t+1} \right = \log \left \frac{\sqrt{1-x}-1}{\sqrt{1-x}+1} \right $															
7	$\sqrt[4]{x} = t$ とおくと $dx = 4t^3 dt$ $= \int \frac{t}{1+t^2} 4t^3 dt = 4 \int \frac{t^4}{t^2+1} dt = 4 \int \left(t^2 - 1 + \frac{1}{t^2+1} \right) dt$ $= 4 \left(\frac{t^3}{3} - t + \tan^{-1} t \right) = \frac{4}{3} x^{\frac{3}{4}} - 4x^{\frac{1}{4}} + 4 \tan^{-1} x^{\frac{1}{4}}$	<table border="1"> <tr><td colspan="2">P149-例7</td></tr> <tr><td>t^2+1</td><td>t^2-1</td></tr> <tr><td></td><td>t^4</td></tr> <tr><td></td><td>t^4+t^2</td></tr> <tr><td></td><td>$-t^2$</td></tr> <tr><td></td><td>$-t^2-1$</td></tr> <tr><td></td><td>1</td></tr> </table>	P149-例7		t^2+1	t^2-1		t^4		t^4+t^2		$-t^2$		$-t^2-1$		1
P149-例7																
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8	$\sqrt[6]{x} = t$ とおくと $dx = 6t^5 dt$ $= \int \frac{6t^5 dt}{t^3(1+t^2)} = 6 \int \frac{t^2+1-1}{t^2+1} dt = 6 \int \left(1 - \frac{1}{t^2+1} \right) dt$ $= 6t - 6 \tan^{-1} t = 6(x^{\frac{1}{6}} - \tan^{-1} x^{\frac{1}{6}})$															
9	$\sqrt{a^2-x^2} = t$ とおくと $x^2 = a^2 - t^2$ $dx = \frac{-t}{x} dt$ $= \int \frac{1}{x \cdot t} \frac{-t}{x} dt = - \int \frac{1}{x^2} dt = \int \frac{1}{t^2-a^2} dt = \frac{1}{2a} \int \left(\frac{1}{t-a} - \frac{1}{t+a} \right) dt$ $= \frac{1}{2a} \log \left \frac{t-a}{t+a} \right = \frac{1}{2a} \log \left \frac{\sqrt{a^2-x^2}-a}{\sqrt{a^2-x^2}+a} \right $															

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$$\begin{aligned} &= -\frac{1}{2} \int \frac{-2x - 1 + 1}{\sqrt{2 - x - x^2}} dx = -\frac{1}{2} \cdot \frac{2}{1} \sqrt{2 - x - x^2} - \frac{1}{2} \int \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 - \left(\frac{1}{2} + x\right)^2}} dx \\ &= -\sqrt{2 - x - x^2} - \frac{1}{2} \sin^{-1} \frac{x + \frac{1}{2}}{\frac{3}{2}} = -\sqrt{2 - x - x^2} - \frac{1}{2} \sin^{-1} \frac{2x + 1}{3} \end{aligned}$$

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