不定積分問題解答

P129

1 $= \frac{1}{2}x^4 + \frac{1}{x} - 2x^{\frac{3}{2}} + 4x$

2 $= \int \frac{\cos 2ax + 1}{2} dx = \frac{1}{2}\left(\frac{1}{2a}\sin 2ax + x\right)$

3 $= \int \frac{1}{\cos ax}\frac{\sin ax}{\cos ax} dx = -\frac{1}{a}\int \frac{(\cos ax)'}{\cos^2 ax} dx = \frac{1}{a\cos ax} = \frac{\sec ax}{a}$

4 $= \frac{1}{3}\int \left(\frac{1}{x^2} - \frac{1}{x^2 + \sqrt{3}^2}\right) dx = \frac{1}{3}\left(-\frac{1}{x} - \frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)$

5 $= \int (a^6 x^{\frac{1}{2}} - 3a^4 x^{\frac{5}{2}} + 3a^2 x^{\frac{9}{2}} - x^{\frac{13}{2}}) dx = \frac{2}{3}a^6 x^{\frac{3}{2}} - \frac{6}{7}a^4 x^{\frac{7}{2}} + \frac{6}{11}a^2 x^{\frac{11}{2}} - \frac{2}{15}x^{\frac{15}{2}}$

6 $= \int \frac{1}{(x+1)^2 + 2^2} dx = \frac{1}{2}\tan^{-1}\frac{x+1}{2}$

7 $= \int \frac{dx}{\sqrt{\frac{a^2}{4} - \left(x^2 - ax + \frac{a^2}{4}\right)}} = \int \frac{dx}{\sqrt{\left(\frac{a}{2}\right)^2 - \left(x - \frac{a}{2}\right)^2}} = \sin^{-1}\frac{x - \frac{a}{2}}{\frac{a}{2}} = \sin^{-1}\frac{2x - a}{a}$

8 $f(x) = x$ とおくと $f'(x) = 1$ $g(x) = \sqrt{a^2 - x^2}$ とおくと $g'^{(x)} = \frac{-x}{\sqrt{a^2 - x^2}}$

$= x\sqrt{a^2 - x^2} - \int \frac{a^2 - x^2 + a^2}{\sqrt{a^2 - x^2}} dx = x\sqrt{a^2 - x^2} - \int \sqrt{a^2 - x^2} dx + \int \frac{a^2}{\sqrt{a^2 - x^2}}$

したがって $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right)$

9 $= \log|f(x)|$

10 $= \frac{1}{2}\int \frac{(3x^2 - 2x + 1)'}{3x^2 - 2x + 1} dx = \frac{1}{2}\log|3x^2 - 2x + 1|$

11 $= -\int \frac{(\cos x)'}{\cos x} dx = -\log|\cos x|$

12 $= \int \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx = \int \frac{\cos^2\frac{x}{2} + \sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx = \int \frac{1 + \tan^2\frac{x}{2}}{2\tan\frac{x}{2}} dx$

$\tan\frac{x}{2} = t$ とおくと $x = 2\tan^{-1} t$ $dx = \frac{2}{1 + t^2} dt$ となるから

$= \int \frac{1 + t^2}{2t} * \frac{2}{1 + t^2} dt = \int \frac{1}{t} dt = \log|t| = \log\left|\tan\frac{x}{2}\right|$

P134

1 $= \frac{1}{5}x^{10} - \frac{3}{7}x^7 + 3x^4 - 3x$

2 $= -\frac{1}{2}e^{-2x} - \frac{1}{4}\cos 4x$

3 $= \int \left(1 - \frac{6}{x} + \frac{9}{x^2}\right)dx = x - 6\log|x| - \frac{9}{x}$

4 $\sqrt{x} + 1 = t$ とおくと $\frac{1}{2\sqrt{x}}dx = dt$ となるから $dx = 2\sqrt{x}dt$

$= \int \frac{t^3}{\sqrt{x}} 2\sqrt{x}dt = 2\int t^3 dt = \frac{1}{2}t^4 = \frac{1}{2}(\sqrt{x}+1)^4$

5 $= \frac{1}{3}\int \left(\frac{1}{x-2} - \frac{1}{x+1}\right)dx = \frac{1}{3}\log\left|\frac{x-2}{x+1}\right|$

6 $= \int \left(2\cos^2 x - \frac{1}{\cos^2 x}\right)dx = \int \left(\cos 2x + 1 - \frac{1}{\cos^2 x}\right)dx = \frac{1}{2}\sin 2x + x - \tan x$

7 $= \int \left(x^2 + ax + a^2 + \frac{a^3}{x-a}\right)dx = \frac{x^3}{3} + \frac{ax^2}{2} + a^2x + a^3\log|x-a|$

$x - a \overline{)\, x^3}$

$x^3 - ax^2$

ax^2

$ax^2 - a^2x$

a^2x

$a^2x - a^3$

a^3

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8 $= \int \log x(\log x)'dx = \frac{1}{2}(\log x)^2$

9 $= \int \cos^6 x\,(-\cos x)'dx = \frac{1}{7}\cos^7 x$

10 $= \int \sin^3 ax \left(\frac{1}{a}\sin ax\right)'dx = \frac{1}{4a}\sin^4 ax$

11 $\sqrt[3]{1+x^2} = t$ とおくと $2xdx = 3t^2dt$ となるから $dx = \frac{3t^2}{2x}dt$

$= \int xt\frac{3t^2}{2x}dt = \frac{3}{2}\int t^3 dt = \frac{3}{8}t^4 = \frac{3}{8}(1+x^2)^{\frac{4}{3}}$

12 $\sqrt[3]{4x+1} = t$ とおくと $4dx = 3t^2dt$ となるから $dx = \frac{3t^2}{4}dt$

$= \int t\frac{3}{4}t^2 dt = \frac{3}{4}\int t^3 dt = \frac{3}{16}t^4 = \frac{3}{16}(4x+1)^{\frac{4}{3}}$

13 $= \int \frac{(a + b\cos x)'}{a + b\cos x}\left(-\frac{1}{b}\right)dx = -\frac{1}{b}\log|a + b\cos x|$

14 $= \frac{1}{3}\int \frac{(1+3\tan x)'}{1+3\tan x}dx = \frac{1}{3}\log|1+3\tan x|$

15 $= \frac{1}{2}\int \frac{(x^2+6x+5)'}{x^2+6x+5}dx = \frac{1}{2}\log|x^2+6x+5|$

16 $= \int \frac{1}{(3x)^2 - 2^2}dx = \frac{1}{12}\int \left(\frac{3}{3x-2} - \frac{3}{3x+2}\right) = \frac{1}{12}\log\left|\frac{3x-2}{3x+2}\right|$

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$$= \int \frac{1}{(x+2)^2+1^2}dx = \frac{1}{1}\tan^{-1}\frac{x+2}{1} = \tan^{-1}(x+2)$$

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$$= \int \frac{1}{\sqrt{2+\frac{1}{4}-(x-1)^2}}dx = \int \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}}dx = \sin^{-1}\frac{x-\frac{1}{2}}{\frac{3}{2}} = \sin^{-1}\frac{2x-1}{3}$$

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$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2+4}}dx + 3\int \frac{1}{\sqrt{x^2+4}}dx = \frac{1}{2}\frac{1}{\frac{1}{2}}\sqrt{x^2+4} + 3\log\left|x+\sqrt{x^2+4}\right|$$

$$= \sqrt{x^2+4} + 3\log\left|x+\sqrt{x^2+4}\right|$$

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$\sqrt{x^2+1} = t$ とおくと $2xdx = 2tdt$ となるから $dx = \frac{t}{x}dt$

$$= \int \frac{1}{xt}\frac{t}{x}dt = \int \frac{1}{t^2-1}dt = \frac{1}{2}\log\left|\frac{t-1}{t+1}\right| = \frac{1}{2}\log\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right| = \frac{1}{2}\log\left|\frac{\left(\sqrt{x^2+1}-1\right)^2}{x^2+1-1}\right|$$

$$= \log\frac{\sqrt{x^2+1}-1}{x}$$

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$f(x) = x$ とおくと $f(x)' = 1$

$g(x) = \sin x$ とおくと $g(x)' = \cos x$

$$= x\sin x - \int \sin x dx = x\sin x + \cos x$$

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$f(x) = x$ とおくと $f(x)' = 1$

$g(x) = \log x$ とおくと $g(x)' = \frac{1}{x}$

$$= x\log x - \int \frac{x}{x}dx = x\log x - x$$

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$f(x) = x^2$ とおくと $f(x)' = 2x$ $f(x) = x$ とおくと $f(x)' = 1$

$g(x) = \frac{1}{a}e^{ax}$ とくと $g(x)' = e^{ax}$ $g(x) = \frac{1}{a}e^{ax}$ とくと $g(x)' = e^{ax}$

$$= \frac{1}{a}x^2e^{ax} - \frac{2}{a}\int xe^{ax}dx = \frac{1}{a}x^2e^{ax} - \frac{2}{a}\left(\frac{1}{a}xe^{ax} - \frac{1}{a}\int e^{ax}dx\right) = \frac{1}{a}e^{ax}\left(x^2 - \frac{2}{a}x + \frac{2}{a^2}\right)$$

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$f(x) = \sqrt{x^2+a^2}$ とおくと $f(x)' = \frac{x}{\sqrt{x^2+a^2}}$

$g(x) = x$ とおくと $g(x)'=1$

$$= x\sqrt{x^2+a^2} - \int \frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}dx = x\sqrt{x^2+a^2} - \int \sqrt{x^2+a^2}dx + \int \frac{a^2}{\sqrt{x^2+a^2}}$$

したがって

$$2\int \sqrt{x^2+a^2}dx = x\sqrt{x^2+a^2} + a^2\log\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \sqrt{x^2+a^2}dx = \frac{1}{2}x\sqrt{x^2+a^2} + \frac{1}{2}a^2\log\left(x+\sqrt{x^2+a^2}\right)$$