Appendix A
Proof of Eq.(11) for a user moving on a curve

A user moves on a bounded curve G at a unit speed. G is randomly placed and intersects C. Assume that the locations of G are independent of $\{\mathbf{w}_i\}_i$.

Because the user moves at a unit speed, the time that the user can use WiFi is equal to the curve length located in the area at which WiFi is available. Even for a bounded curve G, keep the definition of σ_w as the length of $G \cap (C \cap \bigcup_{i=1}^{l} D_i)$ and the definition of $\sigma(X)$ as the length of $G \cap X$ for a convex X. Then, the dynamic P_w is given by Eq. (10).

Let us derive $\int_{C\cap G\neq\emptyset} \sigma(C \cap D_{i_1,i_2,\cdots})dG$ and $\int_{C\cap G\neq\emptyset} \sigma(C)dG$. Let s be the user's location on G. For a convex $X \subseteq C$, define

$$\begin{aligned} I(X,G) &\stackrel{\text{def}}{=} \int_{C\cap G\neq\emptyset} \sigma(X)dG \\ &= \int_{G\cap X\neq\emptyset}\int_{s\in G\cap X} ds\, dG. \end{aligned} \tag{34}$$

Because $\int_{s\in G\cap X} ds$ is the length of G in X and is the time spent in $G\cap X$, $I(X,G)$ is the sum (integral) of the time spent in $G\cap X$ for various locations of G.

By using a method similar to that for the analysis in p. 97 in [19], $I(X,G)$ can be evaluated. Note that $I(X,G)$ is the time spent in $G\cap X$ for various locations of G with a fixed X. Because $I(X,G)$ is invariant under the inversion of motion, it is equivalent to the time spent in $G\cap X$ for various locations of X with a fixed G. That is,

$$I(X,G) = \int_{G\cap X\neq\emptyset}\int_{s\in G\cap X} ds\, dX. \tag{35}$$

For this equation, leave s on G fixed and move X (in translation and in rotation) under the condition $s \in X$ to satisfy $s \in G\cap X$.

$$\begin{aligned} I(X,G) &= \int_{s\in G}\int_{s\in X} dX\, ds = \int_{s\in G} 2\pi\|X\| ds \\ &= \pi|G|\cdot\|X\|. \end{aligned} \tag{36}$$

That is, for $X = C\cap D_{i_1,i_2,\cdots}$,

$$\begin{aligned} &\int_{C\cap G\neq\emptyset} \sigma(C\cap D_{i_1,i_2,\cdots})dG \\ &= \pi|G|\cdot\|C\cap D_{i_1,i_2,\cdots}\|. \end{aligned} \tag{37}$$

Due to Eq. (13) and the equation above,

$$\begin{aligned} &\int_{C\cap G\neq\emptyset,\{C\cap D_i\neq\emptyset\}_{i=1}^{l}} \sigma_w dG\ (dD)^l \\ &= \pi|G|\sum_{m=1}^{l}(-1)^{m-1}\sum_{1\le i_1<\cdots<i_m\le l} \\ &\qquad \int_{\{C\cap D_i\neq\emptyset\}_{i=1}^{l}} \|C\cap D_{i_1,\cdots,i_m}\|(dD)^l. \end{aligned} \tag{38}$$

Due to Eq. (16),

$$\begin{aligned} &\int_{C\cap G\neq\emptyset,\{C\cap D_i\neq\emptyset\}_{i=1}^{l}} \sigma_w dG\ (dD)^l \\ &= \pi|G|\cdot\|C\|\sum_{m=1}^{l}(-1)^{m-1}\sum_{1\le i_1<\cdots<i_m\le l} \\ &\qquad \prod_{i\neq i_1,\cdots,i_m} f(C,D_i)\prod_{j=i_1,\cdots,i_m}(2\pi\|D_j\|). \end{aligned} \tag{39}$$

Similarly, define

$$\begin{aligned} I(C,G) &\stackrel{\text{def}}{=} \int_{C\cap G\neq\emptyset} \sigma(C)dG \\ &= \int_{C\cap G\neq\emptyset}\int_{s\in G\cap C} ds\, dG. \end{aligned} \tag{40}$$

Due to Eq. (36),

$$I(C,G) = \pi|G|\cdot\|C\|. \tag{41}$$

Therefore,

$$\begin{aligned} &\int_{C\cap G\neq\emptyset,\{C\cap D_i\neq\emptyset\}_{i=1}^{l}} \sigma(C)dG\ (dD)^l \\ &= \pi|G|\cdot\|C\|\int_{\{C\cap D_i\neq\emptyset\}_{i=1}^{l}} (dD)^l \\ &= \pi|G|\cdot\|C\|\Pi_{i=1}^{l} f(C,D_i). \end{aligned} \tag{42}$$

Apply Eqs. (39) and (42) to Eq. (10). As a result, we can obtain Eq. (11).

Appendix B
Evaluation of approximation errors in N_h

Two events are ignored in the evaluation of N_h: the event in which $C \subset D_i$ and the event in which there are four intersections between ∂D_i and ∂C.

First, investigate the error regarding $C \subset D_i$. In the numerical examples, $\|C\| > \|D_i\|$ is satisfied. Therefore, the event in which $C \subset D_i$ cannot occur. Thus, at least for the numerical examples in this paper, there is no error caused by this event.

We need to more seriously evaluate the second event. According to Poincare's formula (Eq. (7.11) in [19]),

$$\int_{C\cap D_i\neq\emptyset} n_{C\cap D_i\neq\emptyset} dD_i = 4|C|\cdot|D_i|, \tag{43}$$

where $n_{C\cap D_i\neq\emptyset}$ is the number of intersections between ∂C and ∂D_i under the condition $C\cap D_i\neq\emptyset$. That is,

$$2\Pr(n_{C\cap D_i\neq\emptyset}=2)+4\Pr(n_{C\cap D_i\neq\emptyset}=4) = 4|C|\cdot|D_i|/f(C,D_i). \tag{44}$$

Because $\Pr(n_{C\cap D_i\neq\emptyset}=0)+\Pr(n_{C\cap D_i\neq\emptyset}=2)+\Pr(n_{C\cap D_i\neq\emptyset}=4)=1$,

$$\Pr(n_{C\cap D_i\neq\emptyset}=4) = 2|C|\cdot|D_i|/f(C,D_i)-1+\Pr(n_{C\cap D_i\neq\emptyset}=0). \tag{45}$$

The event $n_{C\cap D_i\neq\emptyset}=0$ is equivalent to that in which $C\subset D_i$ or $D_i\subset C$. As mentioned above in this appendix, $C\subset D_i$ does not actually occur in our examples. Thus, $\Pr(n_{C\cap D_i\neq\emptyset}=0)=\Pr(D_i\subset C)$.

There is no explicit exact formula for $\Pr(D_i\subset C)$ for generic convex C and D_i, but the following formula is applicable. This formula is exact when the greatest radius of the curvature of ∂D_i is less than or equal to the least radius of the curvature of ∂C. On the basis of Eq. (5),

$$\Pr(D_i \subset C) = \frac{2\pi(\|C\| + \|D_i\|) - |C| \cdot |D_i|}{f(C, D_i)}. \tag{46}$$

By substituting Eq. (46) in Eq. (45), $\Pr(n_{C \cap D_i \neq \emptyset} = 4) = 0$. Therefore, as long as Eq. (46) is valid (that is, the assumption for Eq. (46) is satisfied), the error caused by this event is negligible. In the numerical examples, the case in which D_i and C are disks, that in which C has a long perimeter, and that in which C is size satisfy the assumption for Eq. (46). Thus, the error caused by this event is negligible for these cases.

For the other cases in which D_i has a long perimeter or is small, the error may not be negligible. The error is evaluated through simulation under the conditions in Fig. 6, assuming that D_i is a disk-rectangle (Fig. 3). For cases in which D_i has a long perimeter or is small, we find that $\Pr(n_{C \cap D_i \neq \emptyset} = 4)$ is less than 10^{-3}. Thus, the error caused by this event is almost negligible for these cases.