1 Small failure probability

For a ring network of n nodes [see Fig. 1-(a)], let p_i be the failure probability of the i-th link. Assume $p_i = O(1/n^2) \ll 1$ for all i. Let $p(A, B)$ be the probability of disconnection between nodes A and B. Under the assumption of independent link failures, $p(A, B)$ is given by $p(A, B|independent)$.

$$\begin{aligned} & p(A, B|independent) \\ = \; & (1-(1-p_1)\cdots(1-p_k))(1-(1-p_{k+1})\cdots(1-p_m)) \\ = \; & \sum_{i=1}^{k} p_i \sum_{j=k+1}^{m} p_j + o(1/n^4). \end{aligned} \tag{1}$$

When links u, v have a shared part of which the failure probability is $a = O(1/n^2)$, p_u, p_v are given by $p_u = a + b_u + o(1/n^2), p_v = a + b_v + o(1/n^2)$, where $b_u, b_v = O(1/n^2)$ are the failure probabilities of their non-shared parts. Then, $p(A, B)$ is given by $p(A, B|shared)$. When u and v are on different routes between A and B,

$$p(A, B|shared) = a + O(1/n^4), \tag{2}$$

and, when u and v are on a same route,

$$p(A, B|shared) = (a + b_u + b_v + \sum_{i \neq u,v} p_i) \sum_{j=k+1}^{m} p_j + o(1/n^4). \tag{3}$$

Therefore, although $p(A, B|shared)$ can be smaller than $p(A, B|independent)$, the sum of $p(A, B|shared)$ for various A, B is larger than the sum of $p(A, B|independent)$. This is because $\sum_{A,B} p(A, B|shared)$ is minimized when links u, v are consecutive, and because the minimum $\sum_{A,B} p(A, B|shared)$ is $n \times O(1/n^2) = O(1/n)$ and $\sum_{A,B} p(A, B|independent)$ is $n^2 \times O(1/n^4) = O(1/n^2)$.

For the ladder network (combined-ring network) in Fig. 1-(b), $p(A, B)$ is given by the following $p(A, B|independent)$.

$$p(A, B|independent) = \sum_{i=1}^{k1} p_i \sum_{j=k1+1}^{k2} p_j + \sum_{i=k2+1}^{k3} p_i \sum_{j=k3+1}^{k4} p_j + o(1/n^4). \tag{4}$$

When links u, v have a shared part, $p(A, B)$ is given by the following $p(A, B|shared)$. When $1 \leq u, v \leq k1$,

$$p(A, B|shared) = (a + b_u + b_v + \sum_{i \neq u,v} p_i) \sum_{j=k1+1}^{k2} p_j + \sum_{i=k2+1}^{k3} p_i \sum_{j=k3+1}^{k4} p_j + o(1/n^4), \tag{5}$$

when $1 \leq u \leq k1$ and $k1 + 1 \leq v \leq k2$,

$$p(A, B|shared) = a + O(1/n^4), \tag{6}$$

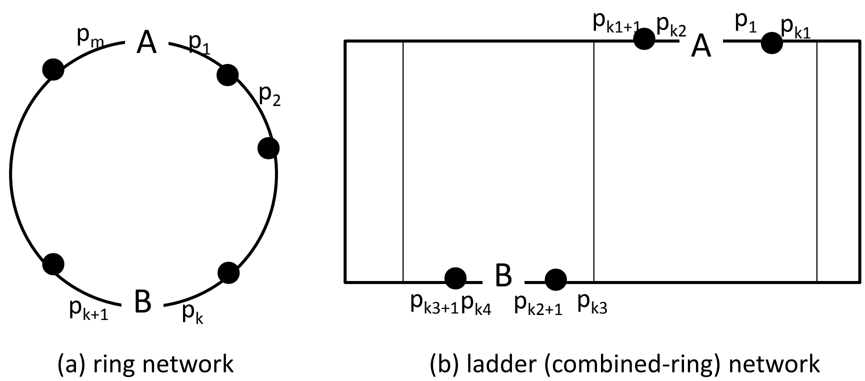

Figure 1: Sensing area models

and when $1 \leq u \leq k1$ and $k2 + 1 \leq v \leq k3$,

$$p(A, B|shared) = O(1/n^4). \tag{7}$$

Similar to the ring network case, the sum of $p(A, B|shared)$ for various A, B is larger than the sum of $p(A, B|independent)$ because the former is $O(1/n)$ and the latter is $O(1/n^2)$.

This result has two implications. When we use a metric of an increasing function of the sum of probabilities of two network nodes disconnecting, (1) a network that has no shared part is better, and (2) the metric value evaluated under the assumption of independent link failure becomes the lower bound of the metric value when taking into account a shared part for a given network.

By using the network N derived by the proposed method for the local problem and the global problem, we can obtain the metric value $V(N)$ when taking into account a shared part in N. For the network N^* achieving the minimum of the metric without assuming independent failure, $V(N^*) \leq V(N)$ because N^* achieves the minimum. We can also obtain the network N' assuming independent failure for the global problem and the metric value $V'(N')$ assuming independent failure. Note that N' achieves the minimum when there is no shared part, that is, the independent failure assumption is valid. Therefore, $V'(N')$ becomes the lower bound of the true metric value (that is, the metric without independence assumption) for any netowork. As a result,$V'(N') \leq V(N^*), V(N)$. Thus,

$$V'(N') \leq V(N^*) \leq V(N). \tag{8}$$

If $V(N) - V'(N')$ is small, $V(N^*) \approx V(N)$. That is, the network N achieves similar performance to N^*.

2 Large failure probability

For a ring network of n nodes [see 1-(a)], assume $1 - p_i = O(1/n^3) \ll 1$ for all i. Let $q(A, B)$ be the probability that nodes A and B can be connected. That is, $q(A, B) = 1 - P(A, B)$. Under the assumption of independent link failures, $q(A, B)$ is given by $q(A, B|independent)$, where $q_i = 1 - p_i = O(1/n^3)$.

$$\begin{aligned} q(A, B|independent) &= 1 - (1 - q_1 \cdots q_k)(1 - q_{k+1} \cdots q_m) \\ &= q_1 \cdots q_k + q_{k+1} \cdots q_m + o(1/n^{3\min(k,m-k)}). \end{aligned} \quad (9)$$

Therefore, $\sum_{A,B} q(A, B|independent) = n \times O(1/n^3) = O(1/n^2)$.

When links u, v have a shared part of which the failure probability is a ($a' = 1 - a = O(1/n^{1.5})$), q_u, q_v are given by $q_u = a'b'_u + o(1/n^3), q_v = a'b'_v + o(1/n^3)$, where $b'_u, b'_v = O(1/n^{1.5})$ are the working probabilities of their non-shared parts. Then, $q(A, B)$ is given by $q(A, B|shared)$. When $1 \le u \le k$ and $k+1 \le v \le m$,

$$q(A, B|shared) = a'(q_1 \cdots q_k b'_u/q_u + q_{k+1} \cdots q_m b'_v/q_v) + o(1/n^{3\min(k,m-k)}), \quad (10)$$

when $1 \le u, v \le k, k > 1$,

$$q(A, B|shared) = q_1 \cdots q_k a' b'_u b'_v/(q_u q_v) + q_{k+1} \cdots q_m + o(1/n^{\min(3k-1.5, 3(m-k))}), \quad (11)$$

and when $k + 1 \le u, v \le m, m - k > 1$,

$$q(A, B|shared) = q_1 \cdots q_k + q_{k+1} \cdots q_m a' b'_u b'_v/(q_u q_v) + o(1/n^{\min(3k, 3(m-k)-1.5)}). \quad (12)$$

Therefore, $\sum_{A,B} q(A, B|shared) = n \times O(1/n^3) = O(1/n^2)$.

That is, $q(A, B|shared) \approx q(A, B|independent) \approx O(1/n^2)$.

Also, for a ladder network, $q(A, B|shared) \approx q(A, B|independent) \approx O(1/n^2)$. Therefore, although $p(A, B|independent)$ is not a lower bound of $p(A, B|shared)$ when a link failure probability becomes large, it is a good approximation of $p(A, B|shared)$. Thus, $V(N^*) \approx V(N)$. That is, the network N achieves similar performance to N^*.

3 Numerical example

We carried out a numerical study to prove the theoretical analysis. We investigated two regions with different geographical maps and earthquake information that have areas of ca. 4500 and 5000 km^2. Region 1 has five earthquake types (one huge and four local ones), while region 2 has four earthquake types (one huge and three local ones). A huge earthquake affects the whole region, and a local one affects only a part of a network.

For each region, we randomly generated 4 types of ladder networks with a different number of nodes and links, each of which includes 20 networks. In each network, a random pair of links is selected to have a shared part. The shared part is the ca. half length of the shorter link. We then calculate the objective function (weighted sum of end-to-end disconnection probabilities assuming equal weights for all earthquake types) in two cases:

- assuming independent failure, which results in V'(N'),
- taking into account dependent failure, which results in V(N).

The differences of these results (in percentage) were calculated. The average difference for each network type is high-lighted in red and is smaller than 2%. This confirms our theoretical analysis. The results are shown in detail in the following tables.

Region 1: one huge earthquake, four local earthquakes

Networks in this region are constructed on the basis of the network shown in Fig. 4 in the paper. For the 12 node-network, the ring part is the same as that of the network in Fig. 4, but the ladder links are randomly chosen. For the 11 node-networks, a random node is removed, the ring part is formed from the 11 nodes, and the ladder links are also randomly chosen.

12 nodes - 14 links				12 nodes - 13 links		
V'(N')	V(N)	Difference (%)		V'(N')	V(N)	Difference (%)
74.1034	74.0002	0.1395		80.7346	78.8090	2.4433
71.5481	69.9248	2.3216		71.7544	70.9557	1.1256
72.9399	70.2672	3.8037		80.5486	79.8092	0.9265
67.4611	64.2518	4.9948		83.4916	83.2925	0.2391
66.4949	65.8857	0.9246		81.1987	80.5393	0.8187
72.3172	69.3887	4.2205		76.2496	75.3587	1.1822
70.3537	69.7969	0.7978		78.3090	77.6616	0.8336
61.8767	60.8950	1.6122		81.2591	77.8460	4.3844
70.9404	70.5831	0.5062		70.6256	70.3191	0.4358
71.6064	70.9070	0.9864		78.7256	78.2698	0.5824
68.7576	67.2849	2.1888		77.5299	76.3811	1.5040
64.2066	63.4415	1.2061		78.7378	77.8351	1.1597
75.2683	73.7991	1.9908		80.7346	78.8090	2.4433
66.7018	65.2635	2.2039		81.2591	77.8460	4.3844
78.6282	75.8682	3.6379		76.7587	75.8532	1.1938
73.5486	73.1543	0.5390		71.2175	69.7387	2.1205
		2.0046				1.6111

11 nodes - 13 links				11 nodes - 12 links		
V'(N')	V(N)	Difference (%)		V'(N')	V(N)	Difference (%)
64.4004	62.6381	2.8134		74.8381	72.7469	2.8746
66.7343	66.0951	0.9672		84.9566	81.7832	3.8803
67.7862	66.3185	2.2131		75.2783	74.2388	1.4002
57.8924	56.9684	1.6220		78.7378	77.8351	1.1597
62.9692	62.1133	1.3780		73.5421	71.8174	2.4016
77.4877	73.7195	5.1115		79.3880	78.9914	0.5021
80.0485	79.4235	0.7869		75.6122	74.4427	1.5710
65.9998	64.1612	2.8656		88.1505	87.0180	1.3015
74.9675	74.3929	0.7725		85.9595	83.8953	2.4604
74.5443	74.3288	0.2899		80.3511	79.4764	1.1006
71.9025	70.0265	2.6790		64.5707	63.7506	1.2863
67.7745	67.4591	0.4676		71.0852	70.6112	0.6712
69.6649	69.2740	0.5642		81.0869	76.8742	5.4799
52.0448	50.9456	2.1576		66.8447	66.1840	0.9982
62.9066	62.6648	0.3859		80.9597	80.4257	0.6639
67.2689	64.4942	4.3022		79.9689	79.1163	1.0777
75.3167	72.5931	3.7518		79.0966	78.6305	0.5928
80.8757	80.8360	0.0491		83.6127	81.2070	2.9624
67.5687	66.9525	0.9204		63.3919	62.6961	1.1099
66.1037	63.9420	3.3808		69.1786	66.1296	4.6106
		1.8739				1.9052

Region 2: one huge earthquake, three local earthquakes

9 nodes - 11 links				9 nodes - 10 links		
V'(N')	V(N)	Difference (%)		V'(N')	V(N)	Difference (%)
76.3377	76.9233	0.7671		81.4277	82.0155	0.7218
78.7750	79.4246	0.8247		78.6254	79.2870	0.8414
76.8407	77.6343	1.0328		80.9375	82.1498	1.4979
72.7183	73.9101	1.6390		79.8555	80.5250	0.8383
77.7553	78.4013	0.8308		81.2218	82.0161	0.9779
75.9576	77.4672	1.9874		80.9375	83.2700	2.8819
78.2753	79.0062	0.9337		80.9375	81.4274	0.6053
76.7562	78.2669	1.9682		79.2812	81.6610	3.0017
78.1156	79.9953	2.4063		77.0059	77.6110	0.7858
73.6240	74.2222	0.8126		79.8555	80.5250	0.8383
73.9959	76.7651	3.7423		80.9375	82.1498	1.4979
71.3440	72.9074	2.1914		79.2812	80.3624	1.3638
76.1885	76.8569	0.8773		79.8555	80.4390	0.7307
76.1795	77.2837	1.4495		77.0059	77.6352	0.8172
80.9133	81.4322	0.6414		81.4277	82.0970	0.8220
79.3149	80.2894	1.2287		79.8607	80.7322	1.0912
75.4439	76.8137	1.8156		78.5286	79.1978	0.8522
80.7946	81.3540	0.6924		78.7268	81.4786	3.4953
78.2149	79.7490	1.9614		79.7103	80.2676	0.6991
75.9880	76.5354	0.7204		79.9292	82.4064	3.0992
		1.4261				1.3730

8 nodes - 10 links				8 nodes - 9 links		
V'(N')	V(N)	Difference (%)		V'(N')	V(N)	Difference (%)
50.6397	51.2572	1.2194		63.1457	65.5177	3.7564
54.6585	55.3285	1.2259		58.0652	58.6961	1.0865
57.4123	58.0872	1.1754		57.8030	58.3326	0.9162
62.3138	63.4105	1.7600		57.8689	58.5397	1.1592
60.4848	61.2281	1.2288		56.8770	58.0560	2.0730
59.8345	61.9568	3.5470		56.8552	58.0326	2.0708
60.5428	61.4303	1.4660		64.1462	64.8812	1.1458
54.4972	55.1081	1.1211		59.4219	60.0141	0.9966
59.1549	60.6407	2.5117		56.8854	57.5387	1.1484
49.2634	50.5502	2.6121		63.6017	64.3682	1.2052
54.9286	56.2056	2.3248		61.0417	61.7184	1.1085
52.3304	53.0611	1.3964		59.6313	60.2840	1.0944
61.4791	64.1267	4.3065		63.7749	64.4457	1.0517
54.4972	55.1834	1.2593		59.3650	59.9213	0.9370
62.2242	63.1700	1.5200		63.0734	63.8013	1.1540
49.9627	50.4769	1.0292		58.5604	60.1499	2.7144
62.2965	62.9753	1.0897		56.9315	58.1435	2.1289
56.3373	56.9600	1.1053		64.2003	64.6977	0.7749
61.7377	62.6704	1.5108		57.8030	58.7459	1.6311
60.8756	63.5457	4.3861		57.8689	59.7046	3.1723
		1.8898				1.5663