Let us discuss the derivation of $\Pr(s \not\leftrightarrow t|U_D)$. Let $N_{cut}(i)$ and $L_{cut}(i)$ be the set of nodes and the set of links in cutset i. In addition, let $Cut(U_D)$ be the set of cutsets for a given U_D. That is, if cutset i is in $Cut(U_D)$, $N_{cut}(i) \subset N_D$ and $L_{cut}(i) \subset S_L(r(s,t)|U_D)$ where $r(s,t)$ is the set of routes between s and t. Let $p_{work}(Cut(U_D))$ be the probability that none of the cutsets in $Cut(U_D)$ actually disconnect s and t. Then,

$$\Pr(s \not\leftrightarrow t|U_D) = 1 - p_{work}(Cut(U_D)). \quad (1)$$

A cutset in $Cut(U_D)$ maintaining connection between s and t can occur when at least a single node or a single link in a cutset in $Cut(U_D)$ is working.

Classify cutsets into exclusive cutset groups (Fig. 1). Let $G_i(Cut(U_D))$ be the i-th cutset group of $Cut(U_D)$ and $n_g(U_D)$ be the number of cutset groups in $Cut(U_D)$. If there exists another cutset i_2 in $G_i(Cut(U_D))$ such that $N_{cut}(i_1) \cap N_{cut}(i_2) \neq \emptyset$ or $L_{cut}(i_1) \cap L_{cut}(i_2) \neq \emptyset$, then cutset i_1 is in $G_i(Cut(U_D))$. That is, in the same cutset group, there is another cutset that has a common node or a common link. On the other hand, if cutset i_1 is in $G_i(Cut(U_D))$ and cutset j_1 is in $G_j(Cut(U_D))$ for $i \neq j$, $N_{cut}(i_1) \cap N_{cut}(j_1) = \emptyset$ and $L_{cut}(i_1) \cap L_{cut}(j_1) = \emptyset$. That is, there is no node or link shared by these two cutsets in different cutset groups. (However, cutsets i_1 and i_2 in $G_i(Cut(U_D))$ may not share a node or a link.)

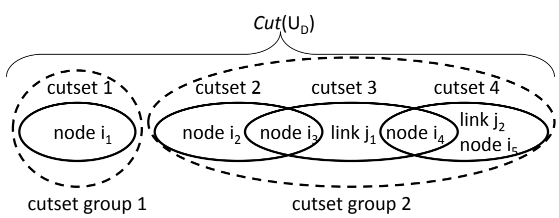

Fig. 1. Definition of cutset group

Let $p_{work}(G_i(Cut(U_D)))$ be the probability that none of the cutsets in $G_i(Cut(U_D))$ will disconnect between s and t. Because cutsets in different cutset groups are exclusive,

$$p_{work}(Cut(U_D)) = \Pi_{i=1}^{n_g(U_D)} p_{work}(G_i(Cut(U_D))). \quad (2)$$

To evaluate $p_{work}(G_i(Cut(U_D)))$, introduce $p_{disc}(Cut(i_1, \cdots, i_k|U_D))$. This is the probability that cutsets $i_1, \cdots, i_k$ in $Cut(U_D)$ actually disconnect s and t, and is given by

$$\begin{aligned} & p_{disc}(Cut(i_1, \cdots, i_k|U_D)) \\ = \; & \Pi_{i \in N_{cut}(i_1) \cup \cdots \cup N_{cut}(i_k)} (1 - p(i)) \\ & \quad \Pi_{i,j \in L_{cut}(i_1) \cup \cdots \cup L_{cut}(i_k)} (1 - p(i,j)). \end{aligned} \quad (3)$$

Because $1 - p_{work}(G_i(Cut(U_D)))$ is the probability that some of the cutsets in $G_i(Cut(U_D))$ disconnect between s and t, it consists of the probability of non-exclusive events: one cutset disconnects s and t, two cutsets disconnect s and t, and so on. By noting that these events are non-exclusive,

$$\begin{aligned} & p_{work}(G_i(Cut(U_D))) \\ = \; & 1 - \{ \sum_{i \in G_i(Cut(U_D))} p_{disc}(Cut(i|U_D)) \\ & - \sum_{i_1 < i_2 \in G_i(Cut(U_D))} p_{disc}(Cut(i_1, i_2|U_D)) \\ & + \sum_{i_1 < i_2 < i_3 \in G_i(Cut(U_D))} p_{disc}(Cut(i_1, i_2, i_3|U_D)) - \cdots \}. \end{aligned} \quad (4)$$

Consider the following examples.

1) A tree network: When $r(s,t)$ is a single route, each cutset in $Cut(U_D)$ is a node in $N_D(r(s,t))$ or a link in $S_L(r(s,t)|U_D)$. These cutsets are exclusive from each other. Thus, $G_i(Cut(U_D))$ has only one element, a node in $N_D(r(s,t))$ or a link in $S_L(r(s,t)|U_D)$, and

$$\begin{aligned} & p_{work}(G_i(Cut(U_D))) = 1 - p_{disc}(Cut(i|U_D)) \\ = \; & \begin{cases} p(j), & \text{if node-}j \text{ is in } G_i(Cut(U_D)) \\ p(j,k), & \text{if } l(j,k) \in S_L(r(s,t)|U_D). \end{cases} \end{aligned}$$

Due to Eqs. (1) and (2),

$$\begin{aligned} & \Pr(s \not\leftrightarrow t|U_D) = 1 - p_{work}(Cut(U_D)) \\ = \; & 1 - \Pi_{i \in N_D} p(i) \Pi_{l(j,k) \in S_L(r(s,t)|U_D)} p(j,k). \end{aligned}$$

2) A ring network: If s or t is in U_D, it is a cutset and included in $Cut(U_D)$. Because there are two routes in the ring network, each cutset, other than s and t, in $Cut(U_D)$ consists of two elements. One is from $N_D(r_1(s,t)) \cup S_L(r_1(s,t)|U_D)$ and the other one is from $N_D(r_2(s,t)) \cup S_L(r_2(s,t)|U_D)$.

For example, assume that node-s and node-1 on $r_1(s,t)$ are in R_G, that $l(s,1), l(1,2), l(s,3) \cap R_G \neq \emptyset$ where $l(s,1)$ and $l(1,2)$ are on $r_1(s,t)$ and $l(s,3)$ is on $r_1(s,t)$, and other nodes or links are in L_G (Fig. 2). Then, $Cut(U_D)$ is divided into $G_1(Cut(U_D))$ and $G_2(Cut(U_D))$, where $G_1(Cut(U_D)) = \{\text{node-s}\}$, $G_2(Cut(U_D)) = \{(\text{node-}1, l(s,3)), (l(s,1), l(s,3)), (l(1,2), l(s,3))\}$.

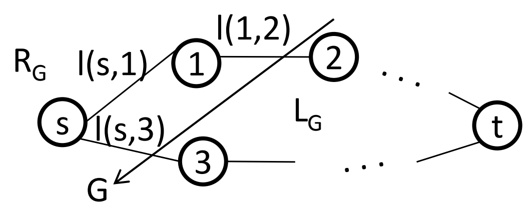

Fig. 2. Example of cutset for ring network

It is clear that $p_{work}(G_1(Cut(U_D))) = p(s)$. In addition,

$$\begin{aligned} & \sum_{i \in G_2(Cut(U_D))} p_{disc}(Cut(i|U_D)) \\ = \; & (1 - p(1))(1 - p(s,3)) + (1 - p(s,1))(1 - p(s,3)) \\ & \quad + (1 - p(1,2))(1 - p(s,3)), \\ & \sum_{i_1 < i_2 \in G_2(Cut(U_D))} p_{disc}(Cut(i_1, i_2|U_D)) \\ = \; & (1 - p(1))(1 - p(s,1))(1 - p(s,3)) \\ & \quad + (1 - p(s,1))(1 - p(1,2))(1 - p(s,3)) \end{aligned}$$

$$
\begin{aligned}
&+(1-p(1))(1-p(1,2))(1-p(s,3)), \\
&\sum_{i_1<i_2<i_3\in G_2(Cut(U_D))} p_{disc}(Cut(i_1,i_2,i_3|U_D)) \\
= \quad &(1-p(1))(1-p(s,1))(1-p(1,2))(1-p(s,3)).
\end{aligned}
$$

According to Eq. (4),

$$p_{work}(G_2(Cut(U_D))) = 1-(1-p(1)p(s,1)p(1,2))(1-p(s,3)).$$

As a result, $\Pr(s \not\leftrightarrow t|U_D) = 1 - p(s)\{1 - (1 - p(1)p(s,1)p(1,2))(1-p(s,3))\}$. On the other hand, for a ring network, $\Pr(s \not\leftrightarrow t|U_D)$ corresponds to $p_{ring}(U_D(r(s,t))$. For this U_D,

$$
\begin{aligned}
&p_{ring}(U_D(r(s,t)) \\
= \quad &\zeta_{12} = 1-p(s)+p(s)\{1-p(s,1)p(1,2)p(1))\}\{1-p(s,3)\}.
\end{aligned}
$$

Thus, $\Pr(s \not\leftrightarrow t|U_D) = \zeta_{12}$.