The Worst Case Cell Arrival Patterns in ATM Networks

Toshiaki TSUCHIYA and Hiroshi SAITO, Members

SUMMARY We introduce the concepts of conservative cell loss ratio (CLR) estimation and worst case cell arrival patterns, and apply them to cell arrival patterns that conform to the generic cell rate algorithm (GCRA). We define new sets of cell arrival patterns which contain the worst case cell arrival patterns for conforming cell arrival patterns. Based on these sets, we propose an upper bound formula using the burst tolerance as well as peak cell rate and sustainable cell rate, and develop a connection admission control method that guarantees cell loss ratio performance satisfying its objective.

key words: ATM, CAC, GCRA, conformance

1. Introduction

The Asynchronous Transfer Mode (ATM) is well known technology for implementing Broadband Integrated Digital Networks. One of major issues in ATM technology is traffic control. In ATM networks, preventive controls are important, although reactive controls such as Explicit Forward Congestion Indication and rate control are mainly used for Available Bit Rate (ABR). The concept of preventive controls is simple. A user notifies the network of traffic characteristics by the source traffic descriptor at a connection set up. The network then decides whether to admit or reject the connection based on the source traffic descriptor, cell delay variation tolerance (CDVT), quality of service (QoS) requirements, network load and the amount of network resources. This decision is called Connection Admission Control (CAC).

If the connection is established after CAC, the network monitors the connection and determines whether it satisfies the source traffic descriptor or not. When the network finds the actual traffic do not conform with the traffic characteristics reported by the source traffic descriptor, the network may impose a penalty on the connection. The conformity is determined by using the Generic Cell Rate Algorithm (GCRA, see Fig.1) [1]. This algorithm is essentially equivalent to the leaky bucket algorithm. The peak cell rate (PCR), the sustainable cell rate (SCR), the burst tolerance (BT) and the cell delay variation tolerance (CDVT) are included in the traffic descriptor. The connection is then tested by GCRA \( \text{T} = \frac{1}{\text{PCR}}, \tau = \text{CDVT} \) and \( \text{GCRA} \ (T=1/\text{SCR}, \tau = \text{BT}+\text{CDVT}) \). We assume that \( \text{CDVT} = 0 \) for simplicity, but our results can be extended to the case in which \( \text{CDVT} \neq 0 \), combined with the results in [3].

A connection which satisfies the conformity requirements is called a compliant connection. The network must guarantee that the QoS objective of the compliant connection is satisfied [1]. In such a situation, an idealistic CAC goal is to keep the QoS satisfactory for all possible conforming cell streams from compliant connections. This objective is often called QoS estimation under the worst case arrival pattern. In practice, the network may provide satisfactory QoS when the network does not experience the worst case arrival pattern, because not all the admitted cell streams have a worst case arrival pattern. However, estimating QoS under the worst case cell arrival pattern is useful because it reduces the QoS monitoring and congestion control.

QoS estimation under the worst case arrival pattern presents a new field in teletraffic theory. The QoS boundaries must be evaluated for the family of arrival processes for which a part of the information is given by a source traffic descriptor or UPC parameters. Saito [2]–[4] considered this bandwidth management issue and introduced the non-parametric approach, since the complete parametric description for the arrival process can not be obtained. His study derived the upper bound of the cell loss ratio (CLR) when the stationary distribution are known for the number of cells arriving during a fixed time period, or for the maximum and average number of cells arriving during a fixed time period. The latter gives a bandwidth that

![Fig. 1 Generic cell rate algorithm (T, \( \tau \)) (continuous state leaky bucket algorithm).](image)

Manuscript received September 22, 1997.
†The authors are with NTT Multimedia Networks Laboratories, Musashino-shi, 180–8585 Japan.
*This paper is an expanded version of a paper presented at GLOBECOM’95, Singapore, November 1995.
accommodates the worst case arrival pattern when the sliding window and the jumping window are used as UPC. (Another formulation and a proof are shown in [2].) However, these results assumed that the burst size may be arbitrarily large. Thus, the proposed bandwidth management results in low utilization when the burst size is finite. Recently, Shioda et al. [5] extended Saito’s results, and clarified the conditions for which these results are valid. Doshi [6] clearly described bandwidth management for the worst case arrival pattern and derived results based mainly on fluid flow approximation. Bonatti et al. [7] also discussed the worst case and derived some formulas. Doshi’s and Bonatti’s formulas are similar to Saito’s, and they must be studied further as the effective bandwidth management techniques for the worst case with a finite burst size. In addition, Yamanaka et al. [8] and Worster [9] gave a non-intuitive example in which a traffic pattern alternating between off-periods and on-periods may not be the worst. In [10], the authors proposed a CLR estimation formula under the worst case arrival pattern in which burst size is finite. This paper is an extended version of [10] and includes newer numerical results, in which the impact of burst size and mixed traffic is considered.

2. Notations and Formulation

Consider a virtual path (VP) of bandwidth \( C \) and with output buffer size \( K \). Here, the VP bandwidth is assumed to be deterministic. That is, the cell transmission of the VP is periodical and its rate is equal to the VP bandwidth when there exists cells waiting for transmission. This VP includes with virtual connections (VCs). (In the remaining of this paper, we consider a VC connection admission in a VP for simplicity. Thus, a connection implies a VC connection in the remaining.) The time axis is assumed to be divided into slots with lengths equal to the cell transmission time. In this paper, all the parameters (such as the PCR \( R_i \) and the SCR \( S_i \) in the \( i \)-th VC’s source traffic descriptor) are transformed into per slot. We also assume that, for each \( i \), the \( i \)-th VC has the same actual average cell rate as its SCR \( S_i \). Here, the “actual” average cell rate means

\[
\lim_{T \to \infty} \frac{\#\{\text{cell arrivals from the } i \text{-th VC in } [0,T]\}}{T}
\]

When the actual average cell rate does not correspond to the specified SCR, further consideration (as in [11]) is needed.

Let \( A_i \) be the cell arrival pattern of the \( i \)-th VC. We assume that all the cell arrival patterns are stationary, ergodic and independent of each other. Consider the cell loss ratio \( CLR(A_1, A_2, \ldots, A_N) \) of the VP at its output buffer when the cell arrival patterns \( A_1, A_2, \ldots, A_N \) are multiplexed. We define the conservative CLR estimation and the worst case cell arrival patterns.

**Definition 1:** A function \( f \) of cell arrival patterns \( \{A_i, i = 1, \ldots, N\} \) of multiplexed VCs is called a conservative CLR estimation of a VP if

\[
f(A_1, \ldots, A_N) \geq CLR(A_1, \ldots, A_N).
\]

**Definition 2:** A cell arrival pattern \( A' \) is called the worst case cell arrival pattern from a VC for a conservative CLR estimation \( f \) in the set \( \Omega \) of cell arrival patterns if

\[
f(A', \vec{A}) \geq CLR(A, \vec{A}) \quad \text{for any } A \in \Omega,
\]

where \( \vec{A} \) denotes any cell arrival pattern from other VCs.

**Definition 3:** Cell arrival patterns \( \{A'_i, i = 1, \ldots, N\} \) are called the worst case cell arrival patterns to a VP for a conservative CLR estimation \( f \) in the sets \( \{\Omega_i, i = 1, \ldots, N\} \) of cell arrival patterns if

\[
f(A'_1, \ldots, A'_N) \geq CLR(A_1, \ldots, A_N)
\]

for any \( A_i \in \Omega_i (i = 1, \ldots, N) \).

Clearly if each cell arrival pattern \( A'_i \) from the \( i \)-th VC is worst case, then \( \{A'_1, \ldots, A'_N\} \) are the worst case cell arrival patterns to a VP.

By combining conservative CLR estimation and worst case cell arrival patterns, we can develop a CAC method. Suppose a new customer requires connection admission when there are \( N \) VCs multiplexed to a VP, and that the arrival process of the \( i \)-th connection is \( A_i \in \Omega_i \) where \( \Omega_i \) is a set of cell arrival patterns conforming to the source traffic descriptor of the \( i \)-th VC. Then, the network calculates \( f(A_1, \ldots, A_N, A_{N+1}) \), where \( A_{N+1} \) is the worst case cell arrival pattern from the \( i \)-th VC for \( i = 1, \ldots, N \), and \( A_{N+1} \) is the worst case cell arrival pattern from a new connection. If the calculated value remains less than the CLR required by each connection, the network accepts the new connection. If not, the new connection is denied by the new network and is lost. With this CAC method, the CLR required by each connection is satisfied as long as the connections maintain their conforming cell arrival patterns in \( \Omega_i, i = 1, \ldots, N + 1 \).

3. Past Results for Conservative CLR Estimation and Worst Case Cell Arrival Patterns

**Theorem 1** (CLR upper bound formula [2]): Let \( X_{A,\gamma} \) be the number of cell arrivals from a VC in the \( \gamma \) time slot and \( p_{A,\gamma}(k) \) be its stationary probability distribution of \( X_{A,\gamma} \) when the cell arrival pattern is \( A \). Set \( p_{A,\gamma} = \{p_{A,\gamma}(0), p_{A,\gamma}(1), \ldots\} \) and define \( D(p_{A,\gamma}; n) = \sum_{k=0}^{\infty}(k - n)^+p_{A,\gamma}(k) \), where \( (x)^+ = \max(0, x) \). Then, we have

\[
D(p_{A,\gamma}; n)/D(p_{A,\gamma}; 0) \geq CLR(A),
\]

where the integer \( n \) is less than \( (K + 1)/2 \).

**Theorem 2** (Shioda and Saito [5]): In the above theorems, we can let \( n \) be less than or equal to \( K + 1 \), when the cell arrival process \( A \) is ergodic.
These two theorems conservatively estimate CLR, but they require detailed information cell arrival distribution \( p_{A_i} \) in a determined number of slots. However, such knowledge is not realistic, especially for the call admission time period. The next theorem provides a CAC method based on the PCR and the SCR by using partial information from them. Let \( \Omega_i(R_i, S_i) \) be the set of conforming cell arrival patterns from the \( i \)-th VC, of which the peak cell rate is \( R_i \) and the sustainable cell rate is \( S_i \). Then, we have

**Theorem 3** (Worst case cell arrival patterns [2]): For the \( i \)-th VC, let \( A_i \in \Omega_i(R_i, S_i), X_{A_i, \gamma} \) and \( p_{A_i, \gamma}(k) \), as in theorem 1. Consider a cell arrival pattern \( A'_i \in \Omega_i(R_i, S_i) \) such that

\[
    p_{A'_i, \gamma}(k) = \begin{cases} 
    1 - S_i / R_i & k = 0 \\
    S_i / R_i & k = \gamma R_i \\
    0 & \text{otherwise}
    \end{cases}
\]  

(see Fig. 2). Then, for any \( A_i \in \Omega_i(R_i, S_i) \), we have

\[
    \frac{D(p_{A'_i, \gamma} \ast \cdots \ast p_{A'_N, \gamma}; n)}{D(p_{A_1, \gamma} \ast \cdots \ast p_{A_N, \gamma}; 0)} \leq \frac{D(p_{A_1, \gamma} \ast \cdots \ast p_{A_N, \gamma}; n)}{D(p_{A_1, \gamma} \ast \cdots \ast p_{A_N, \gamma}; 0)},
\]  

where \( \ast \) denotes the convolution and \( N \) is the number of VCs. In other words, \( A'_i \) is the worst case cell arrival pattern from the \( i \)-th VC in the conforming cell arrival pattern set \( \Omega_i(R_i, S_i) \) and \( \{A'_i, i = 1, \ldots, N\} \) are the worst case cell arrival patterns to a VP in \( \Omega_i(R_i, S_i), i = 1, \ldots, N \).

In theorem 3, the worst case cell arrival pattern \( A'_i \) is such that the cells arrive at \( R_i \) during sufficiently long intervals.

4. The Worst Case Cell Arrival Pattern under the Compliance Property

The worst case cell arrival patterns in theorem 3 use only the PCR and the SCR. Next we try to derive the worst case cell arrival pattern in the set of conforming cell arrival patterns which use the BT as well as the PCR and the SCR. Our method gives a tighter upper bound CLR formula than those in the previous section. We first focus on a particular set of cell arrival patterns. After deriving the worst case cell arrival patterns in that set, we extend the results to the conforming cell arrival patterns.

4.1 Particular Cell Arrival Patterns

Consider a set \( \Omega(R, S, M) \subset \Omega(R, S) \) of cell arrival patterns from a VC and its subset \( \Omega'(R, S, M) \). We assume that any cell arrival pattern \( A \in \Omega(R, S, M) \) has the following properties:

(P1) The cell arrival intervals are either \( r = 1 / R \) or greater than \( W = L r + l \), where \( r, L, l \) are assumed to be integers, \( r > 0, L \geq 0 \) and \( 1 \leq l \leq r \).

(P2) The number of consecutive cell arrivals at rate \( R \) (called a burst) is at most \( M \geq L + 1 \).

Besides (P1) and (P2), we assume that any \( A' \in \Omega'(R, S, M) \subset \Omega(R, S, M) \) has one more property:

(P3) The burst size is always equal to \( M \).

Let \( b_i(i = 1, 2, \ldots, M) \) be the probability that a burst size is \( i \) when the cell arrival patterns are in \( \Omega(R, S, M) \) and set \( b = (b_1, b_2, \ldots, b_M) \). In particular, when the cell arrival pattern is in \( \Omega'(R, S, M) \), we have \( b_i = 0 \) for \( i \leq M - 1 \) and \( b_M = 1 \). We consider the number of arriving cells \( X_{A'} \) in one interval (called a window) of size \( W \) when the cell arrival pattern is \( A \in \Omega(R, S, M) \). Let the distribution of \( X_{A'} \) be \( p(k) = P\{X_{A'} = k\}, k = 0, 1, \ldots, L + 1 \). From the burst size distribution \( b \), we have

\[
    p(k) = \begin{cases} 
    C_1 \left( W - (k - 1)r \right) b_k + \sum_{m=L+1}^{M} 2r b_m & (1 \leq k \leq L - 1) \\
    C_1 \left( W - (L + 1)r \right) b_L + \sum_{m=L+1}^{M} \left( 2r + (m - L - 1)(r - l) \right) b_m & (k = L) \\
    C_1 \sum_{m=L+1}^{M} (m - L) b_m & (k = L + 1)
    \end{cases}
\]  

(see Appendix A), where \( C_1 \) and \( p(0) \) are determined so that \( \sum_k kp(k) = SW \) and \( \sum_k p(k) = 1 \). Similarly, let \( X_{A'} \) be the number of arriving cells in a window when the cell arrival pattern is \( A' \in \Omega'(R, S, M) \) and \( p'(k) = P\{X_{A'} = k\}, n = 0, 1, \ldots, L + 1 \). Then we have

\[
    p'(k) = \begin{cases} 
    2r C_2 & (1 \leq k \leq L - 1) \\
    C_2 \left( 2r + (M - L - 1)(r - l) \right) & (k = L) \\
    C_2 (M - L) l & (k = L + 1)
    \end{cases}
\]  

where \( C_2 \) and \( p'(0) \) are determined so that \( \sum_k k p'(k) = SW \) and \( \sum_k p'(k) = 1 \).
SW and \( \sum_k p'(k) = 1 \) (Fig. 3). Note that we need to consider only a single burst, because we have assumed (P1). Between \( X_A \) and \( X_{A'} \), we have the following order relation

**Theorem 4:** It holds that \( X_A \preceq X_{A'} \) where \( \preceq \) means convex ordering. In other words,

\[
E(X_A - n)_+ \leq E(X_{A'} - n)_+ \quad \text{for all real } n, \tag{9}\]

where \( x_+ = \max(0, x) \).

**Proof:** See Appendix B. \( \square \)

Now, let \( B \) be any cell arrival pattern in some set \( \Omega \) from other VCs and \( q_B(k) = P(X_B = k) \). Theorem 4 then gives

**Theorem 5:** For any integer \( n \), when \( B \in \Omega \), it holds that

\[
\sum_k (k - n)_+ p * q_B(k) / \sum_k k p * q_B(k) \leq \sum_k (k - n)_+ p'(k) * q_B(k) / \sum_k k p'(k) * q_B(k), \tag{10}\]

where * denotes the convolution.

**Proof:** Since convex ordering is closed for convolution, we immediately have \( X_A + X_B \preceq X_{A'} + X_B \). In other words,

\[
\sum_k (k - n)_+ p * q_B(k) \leq \sum_k (k - n)_+ p'(k) * q_B(k), \tag{11}\]

for any integer \( n \). The two Eqs. (10) and (11) are equivalent since \( \sum_k k p * q_B(k) = E(X_A + X_B) \) and \( \sum_k k p'(k) * q_B(k) = E(X_{A'} + X_B) \) are identical. \( \square \)

From theorems 2 and 5, we can conclude that \( A' \) is the worst case cell arrival pattern in the set \( \Omega(R, S, M) \).

4.2 Conforming Cell Arrival Patterns

Now, we can apply theorem 5 to the set \( \Omega(R, S, B) \) of conforming cell arrival patterns to the source traffic descriptor that \( PCR = R \), \( SCR = S \) and \( BT = B \). Consider one conforming cell arrival pattern \( \omega \in \Omega(R, S, B) \), and the number of cell arrivals in a window \( X_\omega \). We denote the \( n \)-th cell arrival time in \( \omega \) as \( a_n \). Then, let us change the pattern \( \omega \) so that the property (P1) holds for the modified pattern as follows (Fig. 4).

**(Step 1)**

**1.1** By moving cell arrival times, change the cell arrival pattern in \([0, a_L] \) so that the consecutive cell arrival intervals become \( r \) and the last cell arrives at time \( a_{L+1} \), where \( r = 1/R \).

**(Step k)**

**k.1** By moving cell arrival times, change the cell arrival pattern in \([a_L, 1, a_{N_k}] \) so that the consecutive cell arrival intervals become \( r \) and the first cell arrives at time \( a_{L+1} \), where \( N_1 = \min\{n : (a_{n+1} - a_L) - r(n - L) \geq W \} \).

**k.2** By moving cell arrival times, change the cell arrival pattern in \([a_{N_k-1} + 1, a_{N_k}] \) so that the consecutive cell arrival intervals become \( r \) and the first cell arrives at time \( a_{N_k-1} + 1 \), where \( N_k \) is determined by

\[
\min\{n : (a_{n+1} - a_{N_k-1}) - r(n - (N_k-1) - L) \geq W \}. \tag{12}\]

Note that after the \( k \)-th step, the modified cell arrival intervals from \( N_{k-1} + 1 \) to \( N_k \) are equal to \( r \) and the interval length between \( a_{N_k} \) (modified) and \( a_{N_k} + 1 \) is equal to or greater than \( W \). Then, consider the possible number of cells \( M = N_k - (N_{k-1} + L) \) gathered in step (k.2). Let \( I = a_{N_k+1} - a_{N_k-1} \) before step (k.2). From (12), we need \( I - rM \geq W \). Since \( \omega \) is conforming, the maximum number of cell arrivals \( M' \) in any interval of length \( I \) is bounded by \( [IS + B] \) where \( [x] \) is the integer part of \( x \). Then, \( M' \leq IS + B \), which corresponds to \( I - rM' \geq I - (IS + B)/R \). Therefore, \( M \) is bounded by \( M' = [I'S + B] \) where \( I' = \min\{I : I - (IS + B)/R \geq W \} \). Consequently, we have

\[
M \leq M' = \left\lceil \frac{WR + B}{R - S} \right\rceil S + B, \tag{13}\]

where \( [x] \) is the smallest integer which is larger than \( x \). Let the modified cell arrival pattern after the \( i \)-th step
be $\tilde{\omega}_i$ and the number of cell arrivals in a window for a pattern $\tilde{\omega}_i$ be $X_{\tilde{\omega}_i}$. Then, we have
\begin{equation}
E(X_{\tilde{\omega}_i} - m)_+ \leq E(X_{\omega_i} - m)_+
\end{equation}
for all $m$ and $i \leq j$ (see Appendix C). Here, let $\tilde{\omega} = \lim_i \tilde{\omega}_i$ and $\tilde{X} = \lim_i X_{\tilde{\omega}_i}$. Then, we have
\begin{equation}
\tilde{\omega} \in \tilde{\Omega}(R, S, M_B),
\end{equation}
where $M_B = L + M'$ and
\begin{equation}
X_{\tilde{\omega}} \leq_{c} \tilde{X}.
\end{equation}
Equation (15) and theorem 5 imply that if we choose some $\omega' \in \tilde{\Omega}(R, S, M_B)$, it will be the worst case arrival pattern in $\tilde{\Omega}(R, S, M_B)$. Furthermore, $\omega'$ is the worst case arrival pattern in $\Omega(R, S, B)$ since (16) holds. Finally, we have

**Theorem 6:** Consider the cell arrival patterns $\omega^i \in \tilde{\Omega}(R_i, S_i, M^i_B), i = 1, \ldots, N$ in which $M^i_B = L^i + M^i$ is determined by
\begin{equation}
M^i \equiv \left[ \begin{array}{c}
\frac{W R_i + B_i}{R_i - S_i} \\
S_i + B_i
\end{array} \right],
\end{equation}
Then, $\{\omega^i \in \tilde{\Omega}(R_i, S_i, M^i_B), i = 1, \ldots, N\}$ are the worst case cell arrival patterns to a VP in the sets $\{\Omega(R_i, S_i, B_i), i = 1, \ldots, N\}$ for the conservative CLR estimation
\begin{equation}
\frac{\sum_k (k - W)_+ p^1 \cdots p^N (k)}{\sum_k k p^1 \cdots p^N (k)},
\end{equation}
where
\begin{equation}
p^i (k) = \left\{ \begin{array}{ll}
2 r_i C^i_1 & (1 \leq k \leq L^i - 1) \\
C^i_1 (2 R_i + (M^i_B - L^i - 1)(1/R_i - 1)) & (k = L^i) \\
C^i_1 (M^i_B - L^i) l^i & (k = L^i + 1).
\end{array} \right.
\end{equation}
Here, the window size $W$ must be equal to or less than the buffer size $+1$, $l^i$ and $L^i$ are non-negative integers, $1 \leq l^i \leq 1/R_i$ and $L^i/R_i + l^i = W$. In (19), $p^i(0)$ and $C^i_1$ are determined so that $\sum_k kp^i(k) = S_i W$ and $\sum_k p^i(k) = 1$.

Note that the worst case cell arrival patterns in theorem 6 give tighter upper bounds than those in theorem 3, because $\tilde{\Omega}_i^j(R_i, S_i, M^i_B) \subset \tilde{\Omega}(R_i, S_i)$ for each $i$. The upper bound based on the theorem 6 is the minimum upper bound when the burst tolerance becomes sufficiently large. This because the upper bound based on the theorem 3 has this property [2] and because that based on the the theorem 6 becomes tighter than that based on the theorem 3.

5. CAC with Conservative CLR Estimation

Theorem 6 provides the following CAC method. Suppose that a new customer requires connection admission when $N$ VCs are already multiplexed to the VP. For each $i \in \{1, \ldots, N\}$, $R_i$, $S_i$ and $B_i$ represent the $i$-th VC's PCR, SCR and BT. Also, $M^i_B$ and $p^i(k)$ are determined by (17) and (19). Then,

1. Receive the PCR $R_{N+1}$, the SCR $S_{N+1}$ and the BT $B_{N+1}$ from the new customer.

2. Estimate the CLR upper bound by
\begin{equation}
\frac{\sum_k (k - W)_+ p^1 \cdots p^{N+1}(k)}{\sum_k k p^1 \cdots p^{N+1}(k)},
\end{equation}
where the window size $W$ is set equal to the VP's output buffer size.

3. Compare the estimated value with the CLR objective.

4. If (20) is equal to or less than the CLR objective, then we accept the new customer's connection as the $N+1$-st VC.

5. If (20) is greater than the objective, then we deny the new customer's connection to avoid degrading the CLR at the VP.

This CAC method guarantees the CLR required by VCs under the condition that each VC maintains its conformity.

6. Numerical Examples

We plot the CLR upper bounds for narrowband VCs (Fig.5) and for broadband VCs (Fig.6) as a function of VP utilization (or the number of VCs). For both figures, VP bandwidth is 150 Mbps, the buffer size for the VP is 128 and the source traffic descriptors of all VCs are the same. In Fig.5, PCR=1.5 Mbps and SCR=5 Mbps. In Fig.6, PCR=15 Mbps and SCR=5 Mbps. The upper bound is a function of the maximum burst size (MBS). Here, MBS is given by the

![Fig. 5 Cell loss ratio (narrowband VCs).](image-url)
positive for $k = 1, \ldots, L - 1$. When PCR is large, however, $p(k) (1 \leq k \leq L - 1)$ cannot be large. This is because $p(k)$ is the same for $k = 1, \ldots, L - 1$, $p(1) + \cdots + p(L-1) < 1$, and $L$ is large. Thus, the MBS has a large impact for low PCR VCs.

Let $CLR*$ be the CLR upper bound when $n$ VCs with various MBSs are multiplexed and $CLR(x)$ be the CLR when $n$ VCs with MBS = $x$ are multiplexed. Figure 5 shows $\sum n_i \log CLR(x_i) / \sum n_i$ is a good estimate of $\log CLR*$ when $n_i$ is the number of VCs with MBS = $x_i$ and $n = \sum n_i$.

7. Conclusion

This paper introduced the concepts of conservative CLR estimation and worst case cell arrival patterns, and applied them to the case in which the patterns conform to GCRMA. With the compliance properties, we defined a new set of cell arrival patterns. This set contained the worst case cell arrival patterns for conforming cell arrivals. As a result, the estimate was tighter than those with existing cell arrival patterns. Therefore, the proposed CAC with CLR estimation better utilizes the VP and can assign an adequate bandwidth as long as all the VCs are conforming. We used a numerical example to demonstrate our method's effectiveness.

References

March 1995.

Appendix A: Number of Cell Arrivals in a Window

For convenience, we consider the case $l = r$, that is, $W = (L + 1)r$. We first focus on one burst with a size of $n \leq L$. In this case, the event $\{X > 0\}$ occurs for $(n - 1)r + W = r(n + L)$ times where $X$ is the number of arriving cells in a window (Fig. A.1). For the first $r$ windows and the last $r$ windows, $\{X = 1\}$ (Fig. A.1). Next, from the $r + 1$-st to the $2r$-th and from the $r(n + L - 2) + 1$-st to the $r(n + L - 1)$-th windows, $\{X = 2\}$ and so on. Consequently, the total window observation results in $\{X = i\}$ for $2r$ times for each $i = 1, \ldots, n - 1$ and $\{X = n\}$ for $(L - n + 2)r = W - (n - 1)r$ times. Similarly, when the burst size is $k \geq L + 1$, we have $\{X = i\}$ for $2r$ times for each $i = 1, \ldots, L$, and $\{X = L + 1\}$ for $(n - L)r$ times. A burst length distribution $\{b_i, \ldots, b_m\}$ gives

$$P\{X = k\} \propto \begin{cases} (W - (k - 1)r)b_k & (1 \leq k \leq L) \\ + \sum_{n=k+1}^{L} 2rb_n & (k = L + 1) \\ + \sum_{n=L+1}^{M} (n - L)r b_n & (n - L + 1) \end{cases}$$

(A.1)

Equation (A.1) corresponds to the special case of (7) when $l = r$.

When $l < r$, we need some modifications, but the argument is straightforward.

Appendix B: Proof of Theorem 5

We only check when $n$ is an integer. First, we calculate $E(X_A - n)_{+}$ in terms of $p(k)$ for $n = 1, 2, \cdots, L$.

$$E(X_A - n)_{+}/C_1 = \sum_{k=n+1}^{L+1} (k - n)p(k)/C_1$$

$$= \sum_{k=n+1}^{L} (k - n)(W - (k - 1)r)b_k$$

$$+ \sum_{k=n+1}^{L} (k - n) \sum_{m=k+1}^{M} 2rb_m$$

$$= (W - nr) \sum_{k=n+1}^{L} (k - n)b_k$$

$$+ \sum_{k=L+1}^{M} ((L - n)r + l)b_k$$

(A.2)

Similarly,

$$E(X_A) = C_2 W \sum_{k=1}^{M} kb_k = SW,$$  \hspace{1cm} (A.3)

$$E(X_{A'} - n)_{+} = C_2 (W - nr)(M - n),$$  \hspace{1cm} (A.4)

and

$$E(X_{A'}) = C_2 WM = SW.$$  \hspace{1cm} (A.5)

We finally have

$$E(X_{A'} - n)_{+} - E(X_A - n)_{+}$$

$$= C_2 (W - nr)(M - n)$$

$$- C_1 (W - nr) \sum_{k=n+1}^{M} (k - n)b_k$$

$$= S(W - nr)(M - n)$$

}\hspace{1cm}(A.6)
\[ -S \left( W - nr \right) \sum_{k=n+1}^{M} (k-n)b_k \sum_{k=1}^{M} kb_k \]
\[ = \left\{ (M-n) \sum_{k=1}^{M} kb_k - M \sum_{k=n+1}^{M} (k-n)b_k \right\} \]
\[ = \left\{ (M-n) \sum_{k=1}^{n} kb_k + n \sum_{k=n+1}^{M} (M-k)b_k \right\} \]
\[ \frac{(W - nr)S}{M \sum_{k=1}^{M} kb_k} \geq 0. \quad (A \cdot 6) \]

Since \( X_A \) and \( X_{A'} \) are at most \( L+1 \), we have \( E(X_A - n)_+ = E(X_{A'} - n)_+ = 0 \) for \( n \geq L+1 \). For \( n \leq 0 \), \( E(X_A - n)_+ = E(X_{A'} - n)_+ = SW - n \).

**Appendix C: Proof of Eq. (14)**

We need to prove the steps (k.1) and (k.2) increase \( X_{\omega_i} \) in convex order. Consider the cell arrival times \( a_{n_1}, \ldots, a_{n+L-1} \), where \( a_n - a_{n-1} \geq W \) and \( r \leq a_i - a_{i-1} \leq W \) for \( i = n, \ldots, n + L - 1 \) (Fig. A-1). Let the arrival time of the \( n \)-th cell move to \( a_{n,1} \) so that \( a_{n+1} - a_{n,1} = r \). This change increases the frequency of \( \{ X = 0 \} \) by \( a_{n+1} - a_n - r \), and decreases the frequency of \( \{ X = 1 \} \) by \( a_{n+1} - a_n - r \). Moreover, several occurrence of \( \{ X = i \} \) should change into \( \{ X = i + 1 \} \) for some \( i = 2, \ldots, L \) (note that \( i \) is not necessarily unique), since the expected value of \( X \) does not change. Those changes of the frequency increases \( X \) in the meaning of convex ordering (simple and straightforward calculation like in Appendix B gives its proof).

Next, let \( a_{n,1} \) and \( a_{n+1} \) move to \( a_{n+2} \) and \( a_{n+1,2} \) so that \( a_{n+2} - a_{n+1,2} = a_{n+1,2} - a_{n,2} = r \). This also increases \( X \) in convex order. Consequently, these changes increase \( X \) in convex order, and construct step (k.1).

Next, consider the cell arrival times \( a_{n+1}, \ldots, a_{n+L+1} \), where \( a_i - a_{i-1} = r \) for \( i = n, \ldots, n + L - 1 \), and \( r \leq a_{n+L+1} - a_{n+L} < W \) (Fig. A-2). Let \( a_{n+L+1} \) move to \( a'_{n+L+1} \) so that \( a'_{n+L+1} - a_{n+L} = r \). The possible decrease in frequencies of the event \( \{ X = L + 1 \} \) is at most \( l \), although the increase in \( \{ X = L + 1 \} \) by \( l \) is certain, and so on. Therefore, this change also increase \( X \) in convex order and repeating this change constructs step (k.2).

**Toshiaki Tsuchiya** received B.S and M.S degrees in information science from Tokyo Institute of Technology in 1990 and 1992 respectively. He has been with NTT Laboratories since 1992. His main research interests are in queueing theory and performance evaluations of communications systems.

**Hirosi Saito** graduated from the University of Tokyo with a B.E degree in Mathematical Engineering in 1981, an M.E degree in Control Engineering in 1983 and received Dr.Eng. in Teletraffic Engineering in 1992. He joined NTT in 1983. He is currently working for teletraffic issues in ATM networks at the NTT Multimedia Networks Laboratories as a Distinguished Technical Member. He received the Young Engineer Award of the Institute of Electronics, Information and Communication Engineers (IEICE) in 1990 and the Telecommunication Advancement Institute Award in 1995. Dr. Saito is a senior member of IEEE, and a member of IFIP WG 7.3, the IEICE, and the Operations Research Society of Japan. He is an ATM forum ambassador. He is the author of the book, Teletraffic Technologies in ATM Networks (Artech House) and a co-author of the book, Basis of Teletraffic Theory and Multimedia Telecommunication Network (IEICE) and so on. He is an editorial board member of Performance Evaluation.